ROTATIONAL MOTION

Moment of Inertia (MOI)

$I = Mr^2$ $kg\,m^2$

$r \rightarrow \perp$ distance from rotational axis

MOI about = ?

About

$I_x = m \cdot y^2$

$I_y = m \cdot x^2$

$I_z = I_y + I_z$

$I_z = mr^2$, $r^2 = x^2 + y^2$

MOI depends on

(i) Mass (M↑ CM↑)
(ii) ⊥ dist.
(iii) Rotational axis
(iv) Mass distribut^n

MOI Calculat^n

→ ∥ axis

$I' = I_{cm} + Md^2$

Applicable to all bodies (2D & 3D)

→ ⊥ axis

Applicable only for 2D

point of intersect^n same hota hai

① x-y plane: $I_z = I_x + I_y$

② x-z plane: $I_y = I_x + I_z$

③ y-z plane: $I_x = I_y + I_z$

Continuous Mass Distribut^n (Body)

$$\int dI = \int dm \cdot r^2$$

① four particles

m → particle ka pass mass hai

② four thin Rod

m → thin rod ka mass

③ Square Lamina

m → ye pure lamina / square ka mass hai

TRANSLATORY

→ Displacement → s
→ velo. → v
→ acc^n → a
→ inertia → mass
→ $F = \frac{dp}{dt} = ma$
→ Linear momentum $\vec{p} = m\vec{v}$
→ $W = \vec{F} \cdot \vec{s}$
→ $P = \vec{F} \cdot \vec{v}$
→ $\vec{W} = \Delta K.E = \frac{1}{2}m[v_f^2 - v_i^2]$

ROTATIONAL

→ Angular displacement $= \theta$
→ Ang. velo. → ω
→ " acc^n → α
→ Moment of Inertia → I
→ $\tau = \frac{dJ}{dt} = I\alpha$
→ Angular momentum J or L = Iω
→ $W_{Rot} = \vec{\tau} \cdot \vec{\theta}$
→ $P_{Rot} = \vec{\tau} \cdot \vec{\omega}$
→ $W_{Rot} = \frac{1}{2}I(\omega_f^2 - \omega_i^2)$

Angular Momentum

(i) Only translatory (x-y plane)

$\vec{J} = \vec{R} \times \vec{p} = \vec{R} \times m\vec{v}$

(ii) Only rotat^n

$J = I\omega$

(iii) If rolling

$\vec{J_0} = \vec{J}_{trans} + \vec{J}_{Rot}$

(A) Thin Rod

(i) MOI about axis ⊥ to the length

$I = \frac{ML^2}{12}$

↳ By continuous mass distribut^n

(ii) MOI about axis passes through either end

$I = \frac{ML^2}{3}$

(B) RING

$I = MR^2$

(C) DISC

$I = \frac{MR^2}{2}$

(B), (C) → 2D

(D) Hollow Sphere

$I = \frac{2}{3}MR^2$

(E) Solid Sphere

$I = \frac{2}{5}MR^2$

(D), (E) → about diameter (3D)

(F) Rectangular Lamina

length → a
Breadth → b
→ Theorem → ⊥ axis

$I_z = \frac{M}{12}[b^2 + a^2]$

↳ $I_x = \frac{Mb^2}{12}$; $I_y = \frac{Ma^2}{12}$

(G) Square Lamina

→ Side − l

Theorem of ⊥ axis

$I_z = \frac{ML^2}{6}$

$I_x = \frac{ML^2}{12}$ $I_y = \frac{ML^2}{12}$

(H) HOLLOW CYLINDER

$I = I_{Ring} + I_{thin\ rod}$

$I_1 = MR^2$
∵ $I_{thin\ rod} = 0$
$I_{ring} = MR^2$

$I_2 = 2MR^2$
∵ $I_{thin\ rod} = 0$
$I_{ring} = MR^2 + MR^2$

Thin rod ka I isliye 0 hai ∵ wo parallel me hai axis ke

$I_3 = \frac{ML^2}{12} + \frac{MR^2}{2}$ (Thin Rod, I Ring)

$I_4 = \frac{ML^2}{3} + \frac{MR^2}{2}$

(I) SOLID CYLINDER

$I = I_{Rod} + I_{Disc}$

$I_1 = \frac{MR^2}{2}$ ∥ $I_{Rod} = 0$, $I_{Disc} = \frac{MR^2}{2}$

$I_2 = \frac{3}{2}MR^2$ ∥ $I_{Rod} = 0$, $I_{Disc} = \frac{3}{2}MR^2$

$I_3 = \frac{ML^2}{12} + \frac{MR^2}{4}$

$I_{Rod} = \frac{ML^2}{12}$ $I_{Disc} = \frac{MR^2}{4}$

$I_4 = \frac{ML^2}{3} + \frac{MR^2}{4}$

$I_{Rod} = \frac{ML^2}{3}$ $I_{Disc} = \frac{MR^2}{4}$

Angular Momentum Conservat^n

Newton → II nd law

↳ $\vec{F} = \frac{d\vec{p}}{dt}$ ($F_{ext} = 0$)

in Rot motion

$\tau = \frac{dJ}{dt}$ | $dJ = 0$ | $J = const.$

→ In absence of external force ($\tau_{ext} = 0$)

$(J_i)_{system} = (J_f)_{system}$

Basic Maths

+i, j, +k, i, k, +j
-i, j, -k, i, -j, k

Clockwise (CW)
→ Anticlockwise (ACW)

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