Motion of System of Particles & Rigid Bodies

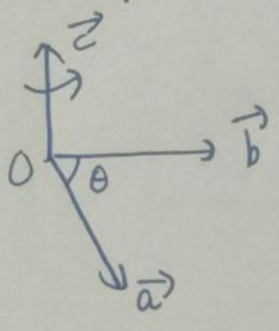
☆ Vector Product :-

1) $\vec{A}\cdot\vec{B} \Rightarrow$ The dot product is defined as :

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$$

↳ Scalar

2) Vector Product (Cross product) :-



$$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\,\cancel{\cos\theta}\,\sin\theta\,\hat{n}$$

Properties :-

(i) $\vec{a}\times\vec{b} \neq \vec{b}\times\vec{a}$

$= -\vec{b}\times\vec{a}$

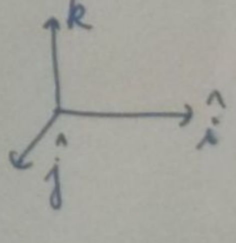
(ii) Reflection $\vec{a} \to -\vec{a}$

$\vec{b} \to -\vec{b}$

$$\vec{a}\times\vec{b} = (-\vec{a})\times(-\vec{b})$$

(iii) $\vec{a}\times\vec{a} = 0$

(iv) $\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{k} = \hat{i}$



(v) $\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$

$\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

☆ Angular Velocity ($\vec{\omega}$) $\Rightarrow$

$$\boxed{\vec{\omega} = \frac{d\vec{\theta}}{dt}}$$

$$\boxed{r_\perp \omega = v}$$

or

$$\boxed{\vec{\omega} \times \vec{r} = \vec{v}}$$

☆ Angular Acceleration :—

$\frac{d\vec{\omega}}{dt} = \vec{\alpha}$, angular acceleration

☆ $v = r\omega$

Diff. both w.r.t. t

$$\frac{dv}{dt} = \cancelto{0}{\frac{dr}{dt}} \omega + \frac{d\omega}{dt} r = r\alpha$$

$$A_t = \frac{dv}{dt} = r\alpha$$

$$\vec{A}_t = \vec{\alpha} \times \vec{r}$$

$$A_r = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = \omega(r\omega)$$

$$\vec{A}_r = \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

★ For a Rigid Body — motion $\Rightarrow$

$$\boxed{\vec{V} = r\omega\,\hat{e}_\theta}$$

$$\vec{A} = \frac{d\vec{V}}{dt} = \frac{d}{dt}\left(\dot{r}\,\hat{e}_r + r\,\dot{v}\,\hat{e}_\theta\right)$$

$$= (\ddot{r} - r\dot{\theta}^2)\,\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{e}_\theta$$

$$= A_r\,\hat{e}_r + A_\theta\,\hat{e}_\theta$$

For a rigid body $\Rightarrow$ $(\dot{r} = 0)$

$$\boxed{\begin{aligned} A_r &= -r\dot{\theta}^2 \\ A_\theta &= r\ddot{\theta} \end{aligned}}$$