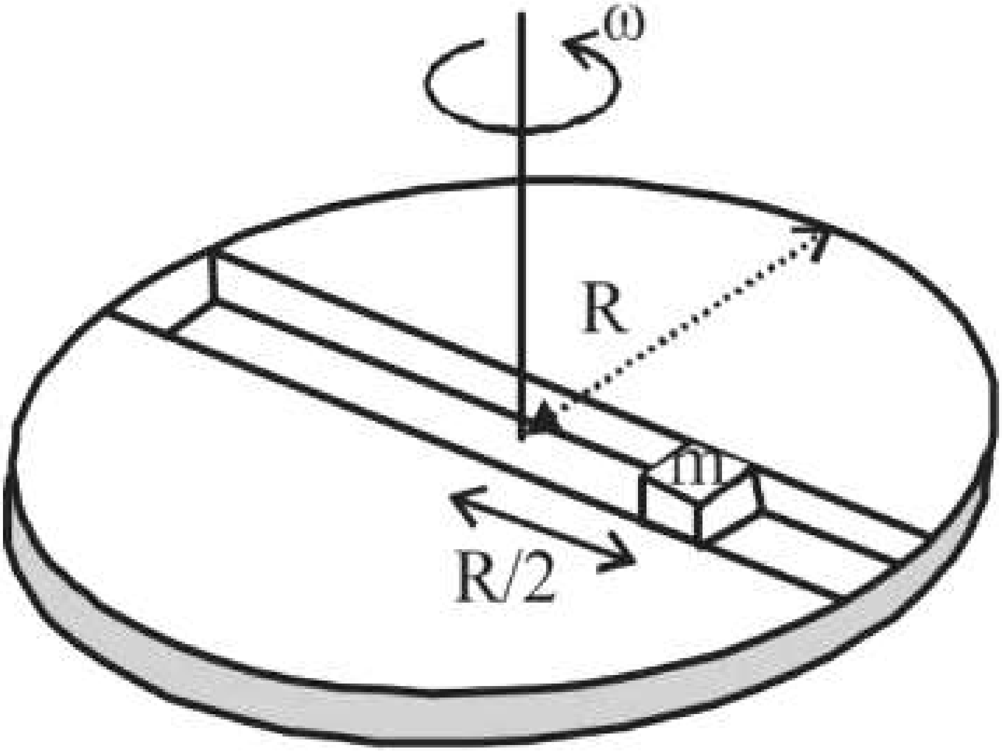
A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of non=inertial frame of reference. The relationship between the force $\vec{F}_{rot}$ experienced by a particle of mass m moving on the rotating disc and the force $\vec{F}_{\in}$ experienced by the particle in an inertial frame of reference is

$\vec{F}_{rot} = \vec{F}_{in} + 2m(\vec{v}_{rot} \times \vec{\omega}) + m(\vec{\omega} \times \vec{r}) \times \vec{\omega}.$

where $\vec{v}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter fo a disc of radius R rotating counter-clockwise with a constant angular speed ω about its vertical axis through its center. We assign a coordinate system with the origin at the center of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the z-axis along the rotation axis $\left(\vec{\omega} = \omega\hat{k}\right)$. A small block of mass m is gently placed in the slot at $\vec{r}(R/2)\hat{i}$ at $t = 0$ and is constrained to move only along the slot.



The distance r of the block at time is

A. (a) $\frac{R}{4}\left(e^{\omega t} + e^{-\omega t}\right)$

B. (b) $\frac{R}{2}\cos\omega t$

C. (c) $\frac{R}{4}\left(e^{2\omega t} + e^{-2\omega t}\right)$

D. (d) $\frac{R}{2}\cos 2\omega t$

Correct Answer - A

Force on the block along slat $= mr\omega^2 = mv\frac{dv}{dr}$

$$\therefore \int_o^v V dv = \int_{R/2}^r \omega^2 r dr \Rightarrow V = \omega\sqrt{r^2 - \frac{R^2}{4}} = \frac{dr}{dt}$$

$$\therefore \int_{R/4}^r \frac{dr}{\sqrt{r^2 - \frac{R^2}{4}}} = \int_o^t \omega dt$$

On solving we get

$$r + \sqrt{r^2 - \frac{R^2}{4}} = \frac{R}{2} e^{wt}$$

or $r^2 - \frac{R^2}{4} = \frac{R^2}{4} e^{2wt} + r^2 - 2r\frac{R}{2} e^{wt}$

$$\therefore r = \frac{R}{4}\left(e^{wt} + e^{-wt}\right)$$