Differential Equation - Class XII

Related Questions with Solutions

Questions

Quetion: 01

The solution of the differential equation $\frac{dy}{dx} = \frac{1}{xy\left[x^2 \sin y^2 + 1\right]}$ is

A. $x^2\left(\cos y^2 - \sin y^2 - 2ce^{-y^2}\right) = 4$

B. $y^2\left(\cos y^2 - \sin y^2 - 2ce^{-y^2}\right) = 2$

C. $x^2\left(\cos x^2 - \sin y^2 - e^{-y^2}\right) = 4$

D. none of these

Solutions

Solution: 01

The given differential equation can be written as $\frac{dx}{dy} = xy\left[x^2 \sin y^2 + 1\right]$

$\Rightarrow \frac{1}{x^3}\frac{dx}{dy} - \frac{1}{x^2}y = y \sin y^2$

This equation is reducible to linear equation, so putting $-1/x^2 = u \Rightarrow \frac{2}{x^3}\frac{dx}{dy} = \frac{du}{dy}$

$\frac{du}{dy} + 2uy = 2y \sin y^2$

The integrating factor of this equation is e^{y^2}.

So, required solution is

$2ue^{y^2} = \int 2y \sin y^2 \cdot e^{y^2} dy + c$

$= \int (\sin t) \cdot e^t dt + c \quad \left(Putting\ t = y^2 \Rightarrow dt = 2ydy\right)$

$= \frac{e^{y^2}}{2}\left(\sin y^2 - \cos y^2\right) + c$

$\Rightarrow 4u = \left(\sin y^2 - \cos y^2\right) + 2ce^{-y^2}$

$\Rightarrow 4 = x^2\left[\cos y^2 - \sin y^2 - 2ce^{-y^2}\right]$

Correct Options

Answer:01

Correct Options: A