Related Questions with Solutions

Questions

Quetion: 01

If $y(t)$ is a solution of $(1+t)\dfrac{dy}{dt} - ty = 1$ and $y(0) = -1$, then $y(1)$ is equal to

A. $\dfrac{-1}{2}$

B. $\dfrac{1}{2}$

C. 1

D. -1

Solutions

Solution: 01

The given differential equation is

$\dfrac{dy}{dt} - \dfrac{t}{1+t}y = \dfrac{1}{1+t}$, which is a linear differential equation

I.F. $= e^{-\int \frac{t}{1+t}dt} = e^{-\int\left(1-\frac{1}{1+t}\right)dt} = e^{-(t-\log(1+t))} = e^{-t} \cdot e^{\log(1+t)} = (1+t)e^{-t}$

$\therefore$ Solution is $y \cdot e^{-t}(1+t) = \displaystyle\int \dfrac{1}{(1+t)}e^{-t}(1+t)dt + C$

$\Rightarrow \quad y \cdot e^{-t}(1+t) = -e^{-t} + C$

$\Rightarrow \quad y = -\dfrac{1}{1+t} + \dfrac{Ce^t}{1+t}$

Given that $y[0] = -1$

$\Rightarrow -1 = -1 + C \Rightarrow C = 0$

$\therefore \quad y = -\dfrac{1}{1+t}$

$\therefore \quad y(1) = -\dfrac{1}{1+1} = -1/2$

Correct Options

Answer:01

Correct Options: A