अवकल समीकरण

परिभाषा :- यदि किसी समी. मे अवकलन के पद उपस्थित है तो उसे अवकल समी. कहते है।

अवकल समी. के प्रकार

1. रैखिक अवकल समी.   2. अरैखिक अवकल समी.

रैखिक अवकल समी. :-

यदि किसी अवकल समी. में y तथा इसके अवकलज का परस्पर गुणा नही होता है तो ऐसी अवकल समी. रैखिक अवकल समी. कहलाता है।

$$\frac{dy}{dx} + 7x = 0 \qquad \frac{d^2y}{dx^2} + 7\frac{dy}{dx} + y = 0$$

अरैखिक अवकल समी. :-

कि यदि किसी अवकल समी. में y तथा इसके अवकलज का परस्पर गुणा होता है तो ऐसी अवकल समी. अरैखिक अवकल समी. कहलाता है।

$$\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right) + y = 0$$

$$\left(\frac{dy}{dx}\right)^2 = 0$$

* कोटि तथा घात   Order and degree :-

$\frac{dy}{dx} \rightarrow 1^{st}$ order

$\frac{d^2y}{dx^2} - 2^{nd}$

$\frac{d^3y}{dx^3} \rightarrow 3^{rd}$

अवकल समी. की कोटि :-
किसी अवकल समीकरण में उपस्थित अधिकतम कोटि का अवकलज पद उस समीकरण की कोटि निर्धारित करता है।

- किसी अवकल समी॰ मे अधिकतम जितनी बार अवकलन हुआ है वह संख्या अवकल समीकरण की कोटि होती है।
- किसी अवकल समीकरण की कोटि सदैव प्राकृत संख्या होती है।

$\frac{dy}{dx} + 7x = 0 \rightarrow 1^{st}$, $\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + y = 0 \rightarrow 2^{nd}$

$\frac{dy}{dx}\left(\frac{d^2y}{dx^2}\right) + 4 = 0 \rightarrow 2^{nd}$, $\left(\frac{dy}{dx}\right)^2 = 0 \rightarrow 1^{st}$

अवकल समी. की घात :-
किसी अवकल समीकरण को बहुपद रूप में परिवर्तित करने के पश्चात अधिकतम अवकलज वाले पद की घात, अवकल समी. की घात कहलाती है।

$\frac{dy}{dx} + 7x = 0 \rightarrow$ 1 घात $\quad \frac{d^2y}{dx^2} + 7\frac{dy}{dx} + y = 0 \rightarrow$ 1 घात

$\frac{dy}{dx}\left(\frac{d^2y}{dx^2}\right) + 4 = 0 \rightarrow$ 1 घात $\quad \left(\frac{dy}{dx}\right)^2 - 0 \rightarrow$ 2 घात

Q $\quad y = x\left(\frac{dy}{dx}\right) + \frac{4}{\left(\frac{dy}{dx}\right)}$

Ans $\quad \left(\frac{dy}{dx}\right) y = x\left(\frac{dy}{dx}\right)^2 + 4$

Order -1
Degree -2

Q $\quad \left[y + x\left(\frac{dy}{dx}\right)^2\right]^{4/3} = x\left(\frac{d^2y}{dx^2}\right)$

Ans $\quad \left[y + x\left(\frac{dy}{dx}\right)^2\right]^4 = x^3\left(\frac{d^2y}{dx^2}\right)^3$

Order -2 $\quad$ Degree -3

Q $\quad \int x^2 dx + \frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0$

Ans $\quad$ Order -2 $\quad$ Degree -1

Q $\quad \frac{d^2y}{dx^2} + \int y\, dx = 0$

Ans $\quad \frac{d^3y}{dx^3} + y = 0 \quad$ Order -3 $\quad$ Degree -1

Q $\quad y = \sin\left(\frac{dy}{dx}\right)$

Ans $\quad \sin^{-1}y = \frac{dy}{dx} \quad$ Order -1
Degree $-$ अपरिभाषित

$y = \sin\left(\frac{dy}{dx}\right)$
प्रसार करने पर Power निरन्तर बढेगी तो Degree अपरिभाषित है।

आंशिक अवकल समीकरण :-
यदि किसी समीकरण में आंशिक अवकलन के पद उपस्थित है तो उसे आंशिक अवकल समीकरण कहते हैं।

$\frac{\partial y}{\partial x} + 4 = 0$

अवकल समीकरण का हल :- तीन प्रकार

1 व्यापक हल :-

किसी अवकल समीकरण में चर का वह मान जो दि गयी समीकरण को सन्तुष्ट करता है तथा जिसमें अवकल समीकरण की कोटि के समान स्वेच्छ अचर उपस्थित है अवकल समीकरण का व्यापक हल कहलाता है।

2 विशिष्ट हल :-

किसी अवकल समी. के व्यापक हल में उपस्थित स्वेच्छ अचरों को कोई विशिष्ट मान प्रदान करने पर प्राप्त हल, अवकल समी. का विशिष्ट हल कहलाता है

Example :- $\frac{d^2y}{dx^2} = 0$

$\frac{dy}{dx} = C_1 x \Rightarrow y = C_1 x + C_2 \rightarrow$ व्यापक हल

$\left.\begin{array}{l} y = 2x+3 \\ y = x \end{array}\right]$ — विशिष्ट हल

3 विचित्र हल

Q. (h, k) केन्द्र वाले वृत्त निकाय की अवकल समी. की कोटि ज्ञात करो

Ans $(x-h)^2 + (y-k)^2 = r^2$ { इसमे h व k तो fix है परन्तु r को Change कर सकते है └ स्वेच्छ अचर

कोटि — 1

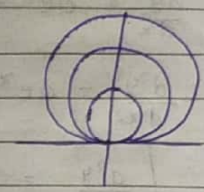
Q r त्रिज्या वाले वृत्त निकाय की अवकल समी. की कोटि ज्ञात करो

Ans $(x-h)^2 + (y-k)^2 = r^2$ r = fix

h व k → स्वेच्छ अचर

कोटि — 2

Q उन सभी वृत्तों की अवकल समी. ज्ञात करो जो मूल बिन्दु से गुजरते है तथा जिनके केन्द्र y-अक्ष पर स्थित है?



Ans $(x-0)^2 + (y-h)^2 = h^2$

$x^2 + y^2 - 2hy + h^2 = h^2$

$x^2 + y^2 - 2hy = 0$

$2x + 2y\frac{dy}{dx} - 2h\frac{dy}{dx} = 0$

$2x + 2y\frac{dy}{dx} = 2h\frac{dy}{dx}$

$h = \frac{\left[x + y\frac{dy}{dx}\right]}{dy/dx}$ — 1

$x^2 + y^2 - 2y\left[x + y\frac{dy}{dx}\right] \Big/ \frac{dy}{dx} = 0$

$x^2\frac{dy}{dx} + y^2\frac{dy}{dx} - 2xy - 2y^2\frac{dy}{dx} = 0$

$x^2\frac{dy}{dx} - y^2\frac{dy}{dx} - 2xy = 0$

Q $x = A\cos(nt + \alpha)$

$x = A[\cos nt \cos\alpha - \sin nt \sin\alpha)$

$x = A_1 \cos nt + A_2 \sin nt$ $[A_1 = A\cos\alpha, A_2 = -A\sin\alpha$

$\frac{dx}{dt} = -An\sin[nt + \alpha)$

$\frac{d^2x}{dt^2} = -An^2\cos(nt + \alpha)$

$\frac{d^2x}{dt^2} = -n^2 . x \Rightarrow \boxed{x'' = -n^2 x}$ — SHM

Order = 2

Q $y = ae^{x} + be^{-x}$

$\frac{dy}{dx} = ae^{x} - be^{-x}$

$\frac{d^2y}{dx^2} = ae^{x} + be^{-x}$

$\boxed{\frac{d^2y}{dx^2} = y}$

Q $y = ae^{3x} + be^{-3x} + ce^{x}$

$\frac{dy}{dx} = 3ae^{3x} - 3be^{-3x} + ce^{x}$

$\frac{d^2y}{dx^2} = 9ae^{3x} + 9be^{-3x} + ce^{x}$

$\frac{d^3y}{dx^3} = 27ae^{3x} - 27ae^{-3x} + ce^{x}$

$\frac{d^3y}{dx^3} = \cancel{dy}\, 9\left[\frac{dy}{dx} - ce^{x}\right] + ce^{x}$

$\frac{d^3y}{dx^3} = 9\frac{dy}{dx} - 9ce^{x} + ce^{x}$

$\frac{d^3y}{dx^3} = 9\frac{dy}{dx} - 8ce^{x}$

Q $Ax^2 + By^2 = 1$

Ans $y^2 = \frac{1 - Ax^2}{B}$

$y^2 = \frac{1}{B} - \frac{A}{B}x^2$

$2y\frac{dy}{dx} = \frac{-A}{B} \cdot 2x \Rightarrow \frac{-A}{B} = \frac{y}{x}\frac{dy}{dx}$

$2\left(\frac{dy}{dx}\right)^2 + 2y\frac{d^2y}{dx^2} = -\frac{A}{B}2$

$\left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2} = \frac{y}{x}\frac{dy}{dx}$

$xy\frac{d^2y}{dx^2} + \cancel{y}x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

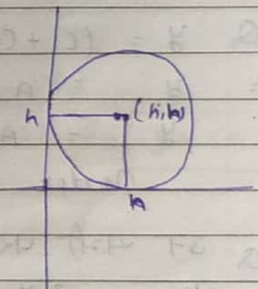
Q. प्रथम चर्तुथांश में स्थित उस वृत्त निकाय के अवकल समीकरण जो निर्देशी अक्षों को स्पर्श करती है तो उसकी कोटि क्या होगी

Ans :- $(x-h)^2 + (y-h)^2 = h^2$

$x^2 + h^2 - 2xh + y^2 + h^2 - 2yh = h^2$

$x^2 + y^2 - 2xh - 2yh = -h^2$

कोटि $= 1$



Q $\left\{\frac{d^2y}{dx^2}\right\}^{-3/2} = 0$ दो बार समाकलन करने पर

$\left(\frac{d^2y}{dx^2}\right)^{1/2} = C_1x + C_2$

Order $= 4$

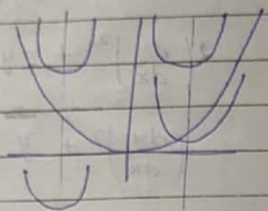
degree $= 1$

Q xy समतल मे स्थित रेखाओ की अवकल समीकरण क्या होगी

Ans $y = mx + c$

$\frac{dy}{dx} = m \Rightarrow \boxed{\frac{d^2y}{dx^2} = 0}$

Q y-अक्ष के समान्तर अक्ष वाले सभी परवलयो का अवकल समी. क्या होगा



Ans $(x-h)^2 = 4a(y-k)$

$x^2 + h^2 - 2xh = 4ay - 4ak$

$2x - 2h = 4a\frac{dy}{dx}$

$2 = 4a\frac{d^2y}{dx^2}$

$\frac{d^3y}{dx^3} = 0$

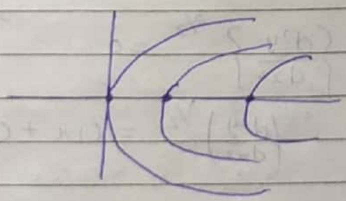
Q $y = (C_1 + C_2)\cos(x + C_3) - C_4 e^{x + C_5}$

Ans = $y = A\cos(x + C_3) - C_4 e^x \cdot e^{C_5}$

$y = A\cos(x + C_3) - Be^x$

Order $= 3$

Q उन सभी परवलयो का अवकल समी. ज्ञात करो जिसके अक्ष, x-अक्ष है।



Ans = $y^2 = 4a(x-h)$

Order $= 2$

$y^2 = 4ax - 4ah$

$2y\frac{dy}{dx} = 4a$

$2\left(\frac{dy}{dx}\right)^2 + 2y\frac{d^2y}{dx^2} = 0 \Rightarrow y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

Q $K\frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{1/5}$ Order = ? & degree = ?

Ans $K^5\left(\frac{d^2y}{dx^2}\right)^5 = 1 + \left(\frac{dy}{dx}\right)^2$

Order = 2. Degree = 5

* Sepration of variable :- चर पृथक्करण

$\frac{dy}{dx} = \phi(x, y)$

$\frac{dy}{dx} = \frac{f(x)}{g(y)}$

$\int g(y)\,dy = \int f(x)\,dx$

$\int g(y)\,dy = \int f(x)\,dx + C$

Q $x\,dy - y\,dx = 0$

$\frac{dy}{y} - \frac{dx}{x} = 0$

$\log y - \log x = \log c$

$\log\left(\frac{y}{x}\right) = \log c \Rightarrow \frac{y}{x} = c \Rightarrow \boxed{y = cx} \rightarrow$ रेखा

Q $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

$e^y dy = e^x + x^2 \Rightarrow e^y = e^x + \frac{x^3}{3} + C$

Q $\frac{dy}{dx} = e^{x+y} + x^2 e^y$

$e^{-y} dy = (e^x + x^2)\,dx \quad -e^{-y} = e^x + \frac{x^3}{3} + C$

Q $(1-x^2)(1-y)\,dx = xy(1+y)\,dy$

$\frac{1-x^2}{x}\,dx = \frac{y(1+y)}{1-y}\,dx$

$\left(\frac{1}{x}-x\right)dx = \left(-y-2+\frac{2}{1-y}\right)dy$

$\log x - \frac{x^2}{2} = -\frac{y^2}{2} - 2y - 2\log(1-y) + c$

Q $x\frac{dy}{dx} = y-1$, $y(0) = 2$ हलों की संख्या = ?

$\frac{dy}{y-1} = \frac{dx}{x}$

$\log(y-1) = \log(x) + \log c$

$y+1 = cx$

$y = cx+1$

$y=0 \Rightarrow x=2$

$0-1 = 2c$

$c = -\frac{1}{2}$

$\boxed{y-1 = -x/2}$ हलों की संख्या – 1

Q. $(x+y)(dx-dy) = dx+dy$

Ans $dx-dy = \frac{dx+dy}{x+y}$

$x-y = \log(x+y) + c$ Ans

Q. $\frac{dy}{dx} = e^{x+y}$ $x=1$ पर $y=-1$ है
$x=-1$ पर $y=?$

$\frac{dy}{dx} = e^x . e^y$

$dy\, e^{-y} = e^x dx$

$-e^{-y} = e^x + c$

$-e^{-1} = e^1 + c$

$c = -e - \frac{1}{e}$

$-e^{-y} = e^x - e - \frac{1}{e}$

$x = -1$ पर

$-e^{-y} = e^{-1} - e - \frac{1}{e} \Rightarrow -e^{-y} = \frac{1}{e} - e - \frac{1}{e}$

$y = -1$ Ans

Q $\frac{dy}{dx} = \frac{x(\cancel{x}\log x + 1)}{\sin y + y\cos y}$

$(\sin y + y\cos y)\,dy = (x\log x + x)\,dx$

$-\cos y + y\sin y - \int \sin y\,dy = x^2\log x - \int\left(\frac{1}{x}\,x^2\right)dx + \frac{x^2}{2}\,dx$

$-\cos y + y\sin y + \cos y = x^2\log x - \frac{x^2}{2} + \frac{x^2}{2} + c$

$y\sin y = x^2\log x + c$

Q $\frac{dy}{dx} = \frac{1+y^2}{1+x^2} \Rightarrow \frac{dy}{1+y^2} = \frac{dx}{1+x^2}$

$\tan^{-1}(y) = \tan^{-1}(x) + \tan^{-1}(c)$

$\tan^{-1}(y) - \tan^{-1}(x) = \tan^{-1} c$

$\tan^{-1}\left(\frac{y-x}{1+xy}\right) = \tan^{-1} c$

$\frac{y-x}{1+xy} = c \Rightarrow y-x = c(1+xy)$

Q $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$

$\frac{dy}{\sqrt{1-y^2}} + \frac{dx}{\sqrt{1-x^2}} = 0$

$\sin^{-1}y + \sin^{-1}x = \sin^{-1}c$

Q $x\sqrt{1+y^2}\,dx + y\sqrt{1+x^2}\,dy = 0$

$\frac{x}{\sqrt{1+x^2}}dx + \frac{y}{\sqrt{1+y^2}}dy = 0$

$\left\{ \frac{d}{dx}\sqrt{1+x^2} = \frac{1 \cdot 2x}{2\sqrt{1+x^2}} \right.$

$\sqrt{1+x^2} + \sqrt{1+y^2} = C$

$\frac{d}{dx}\sqrt{1+x^2} = \frac{x}{\sqrt{1+x^2}}$

Q. $a\left(x\frac{dy}{dx} + 2y\right) = xy\frac{dy}{dx}$

Ans $ax\frac{dy}{dx} + 2ay = xy\frac{dy}{dx}$

$2ay = x(y-a)\frac{dy}{dx}$

$\frac{2a}{x}dx = \frac{y-a}{y}dy$

$2a\log x = y - a\log(y) + c$

Q $y - x\frac{dy}{dx} = 3\left(1 + x^2\frac{dy}{dx}\right)$

$y - x\frac{dy}{dx} = 3 + 3x^2\frac{dy}{dx}$

$y - 3 = (3x^2+x)\frac{dy}{dx}$

$\frac{dx}{x(3x+1)} = \frac{dy}{y-3}$

$\left(\frac{1}{x} - \frac{3}{3x+1}\right)dx = \frac{dy}{y-3}$

$\log x - \log(3x+1) = \log(y-3) + \log c$

$\frac{x}{3x+1} = c(y-3)$

Q. $\sec^2 x \tan y\,dy + \sec^2 y \tan x\,dx = 0$

Ans $\frac{\tan y}{\sec^2 y}dy + \frac{\tan x}{\sec^2 x}dx = 0$

$\sin y \cos y\,dy + \sin x \cancel{\sin} \cos x\,dx = 0$

$\frac{\sin 2y}{2}dy + \frac{\sin 2x}{2}dx = 0$

$-\frac{\cos 2y}{4} - \frac{\cos 2x}{4} = c$

$\cos 2x + \cos 2y = -4c$

$\cos 2x + \cos 2y = A$

* समघात अवकल समीकरण :-

$y = vx$ | $x = vy$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ | $\frac{dx}{dy} = v + y\frac{dv}{dy}$

Q $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$

$y = vx$

$v + x\frac{dv}{dx} = \frac{x^2(1+v^2)}{x^2(2v)} \Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{2v} - v$

$x\frac{dv}{dx} = \frac{1-v^2}{2v} \Rightarrow \frac{2v}{1-v^2}dv = \frac{dx}{x}$

$-\log(1-v^2) = \log(x) + \log C$

$\frac{1}{1-v^2} = xc$

$V = \frac{y}{x}$

$\frac{1}{\frac{1-y^2}{x^2}} = cx \Rightarrow \frac{x}{x^2-y^2} = c$

Q. $x^2 dy + y(x+y)\,dx = 0$

Ans $\frac{dy}{dx} = -\frac{y(x+y)}{x^2}$

$x\frac{dv}{dx} = \underline{-v(1+v)} - v$

$x\frac{dv}{dx} = -v - v^2 - v = -(v^2+2v)$

$-\frac{dx}{x} = \frac{dv}{v(v+2)}$

$-\frac{dx}{x} = \left(\frac{1}{2v} - \frac{1}{2(v+2)}\right) dv$

$-\log x + \log c = \frac{1}{2}\left[\log v - \log(v+2)\right]$

$-\log x^2 + \log c = \log\left(\frac{v}{v+2}\right)$

$\frac{v}{v+2} = \frac{c}{x^2} \Rightarrow \frac{y/x}{\frac{y+2x}{x}} = \frac{c}{x^2}$

$\frac{y}{y+2x} = \frac{c}{x^2}$

Q. $(1 + e^{x/y})\,dx + e^{x/y}\left(1 - \frac{x}{y}\right) dy = 0$

$\frac{dx}{dy} = -\frac{e^{x/y}(1 - x/y)}{1+e^{x/y}}$

$x = vy$

$\frac{dx}{dy} = v + y\frac{dv}{dy}$

$v + y\frac{dv}{dy} = \frac{-e^v(1-v)}{1+e^v}$

$y\frac{dv}{dy} = \frac{-e^v + ve^v - v - ve^v}{1+e^v}$

$y\frac{dv}{dy} = \frac{-(v+e^v)}{1+e^v}$

$\frac{(1+e^v)\,dv}{(v+e^v)} = \frac{-dy}{y}$

$\log(v+e^v) = -\log(y) + \log c$

$v + e^v = \frac{c}{y}$

$\frac{x}{y} + e^{x/y} = \frac{c}{y}$

$\boxed{x + y e^{x/y} = c}$

Q $x\frac{dy}{dx} = y(\log y - \log x + 1)$

$\frac{dy}{dx} = \frac{y}{x}\left[\log\left(\frac{y}{x}\right) + 1\right]$

$x\frac{dv}{dx} = v[\log v + 1] - v$

$x\frac{dv}{dx} = v\log v \Rightarrow \frac{dv}{v\log v} = \frac{dx}{x}$

$\log(\log v) = \log x + \log c$

$\log v = cx$

Q $\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$

$x\frac{dv}{dx} = v + \tan v - v \Rightarrow \frac{dv}{\tan v} = \frac{dx}{x}$

$\log(\sin v) = \log x + \log c \Rightarrow \sin v = cx$

$\sin y/x = cx$

$\log y$ या $\log x$ = समघात फलन नहीं

Q. $\frac{dy}{dx} + \frac{x^3 + 3y^2}{3x^2 + y^2} = 0$

$v + x\frac{dv}{dx} + \frac{1 + 3v^2}{3 + v^2} = 0$

$x\frac{dv}{dx} + \frac{1 + 3v^2 + 3v + 1}{(3+v^2)} = 0$

$x\frac{dv}{dx} + \frac{(1+v)^3}{3+v^2} = 0$

$\frac{dx}{x} + \frac{(3+v^2)}{(1+v)^3}dv = 0$

$\frac{dx}{x} + \left\{\frac{1}{v+1} - \frac{2}{(v+1)^2} + \frac{4}{(v+1)^3}\right\}dv = 0$

$\log x + \log(v+1) + \frac{2}{(v+1)} - \frac{2}{(v+1)^2} = c$

$\log x + \log\left(\frac{y}{x}+1\right) + \frac{2x}{(y+x)} - \frac{2x^2}{(y+x)^2} = c$ Ans

$\frac{3+v^2}{(v+1)^3} = \frac{A}{v+1} + \frac{B}{(v+1)^2} + \frac{C}{(v+1)^3}$

$3+v^2 = A(v+1)^2 + B(v+1) + C$

$C = 4 \quad (v = -1)$

$A = 1$ [v^2 के गुणांक]

$0 = 2A + B$ [v के गुणांक]

$B = -2$

Q. $x^3\frac{dy}{dx} = y^3 + y^2\sqrt{y^2 - x^2}$

$\frac{dy}{dx} = \frac{y^3 + y^2\sqrt{y^2-x^2}}{x^3}$ $\qquad y = vx$

$x\frac{dv}{dx} = v^3 + v^2\sqrt{v^2-1} - v$

$x\frac{dv}{dx} = v^3 + v^2\sqrt{v^2-1} - v$

$= v(v^2-1) + v^2\sqrt{v^2-1}$

$= v\sqrt{v^2-1}\left[\sqrt{v^2-1} + v\right]$

$\frac{dx}{x} = \frac{dv}{v\sqrt{v^2-1}\left[\sqrt{v^2-1} + v\right]}$

हर का परिमेयकरण

$\frac{dx}{x} = \frac{dv \left[\sqrt{v^2-1} - v\right]}{v\sqrt{v^2-1}\left[\sqrt{v^2-1}+v\right]\left[\sqrt{v^2-1} - v\right]}$

$\frac{dx}{x} = \frac{\left(\sqrt{v^2-1} - v\right)dv}{v\sqrt{v^2-1}\left[v^2 - 1 - v^2\right]}$

$\frac{dx}{x} = \frac{\left(v - \sqrt{v^2-1}\right)dv}{v\sqrt{v^2-1}}$

$\frac{dx}{x} = \frac{1}{\sqrt{v^2-1}}dv - \frac{1}{v}dv$

Q $(y^3 - 2x^2y)dx + (2xy^2 - x^3)dy = 0$

$\frac{dy}{dx} = -\frac{y^3 - 2x^2y}{2xy^2 - x^3} = \frac{2x^2y - y^3}{2xy^2 - x^3}$

$x\frac{dv}{dx} = \frac{2v - v^3}{2v^2 - 1} - v$

$x\frac{dv}{dx} = \frac{2v - v^3 - 2v^3 + v}{2v^2 - 1} = \frac{-3v^3 + 3v}{2v^2-1}$

$x\frac{dv}{dx} = \frac{3v(1-v^2)}{2v^2-1} = \frac{3v(v+1)(v-1)}{2v^2-1}$

$\frac{dx}{x} = \frac{dv\,(2v^2-1)}{3v(v+1)(v-1)}$

$\frac{1}{3}\left[-\frac{1}{v}+\frac{1}{2(1-v)}-\frac{1}{2(1+v)}\right]dv = \frac{dx}{x}$

$\frac{1}{3}\left[-\log v + \frac{1}{2}\log(1-v) - \frac{1}{2}\log(1+v)\right] = \log x + \log c$

$-\log v^3 - \log(1-v)^6 - \log(1+v)^6 = \log x + \log c$

$\left[v^3(1-v)^6(1+v)^6\right] = cx$

$-\left[\left(\frac{y}{x}\right)^3\left(1-\frac{y}{x}\right)^6\left(1+\frac{y}{x}\right)^6\right] = cx$

Q. $y^2 = x^2 + 2xy\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{y^2-x^2}{2xy}$

$x\frac{dv}{dx} = \frac{v^2-1}{2v} - v$

$x\frac{dv}{dx} = \frac{-(1+v^2)}{2v}$

$\frac{2v\,dv}{1+v^2} = -\frac{dx}{x}$

$\log(1+v^2) = -\log x + \log c$

$1+v^2 = \frac{c}{x}$

$1+\frac{y^2}{x^2} = \frac{c}{x} \Rightarrow x^2+y^2-cx=0 \rightarrow$ वृत्त का समीकरण

Q m प्राकृत संख्या है $y = e^{mx}$

$D^3y - 3D^2y - 4Dy + 12y = 0$ $\boxed{e^{mx} \neq 0}$

$(m^3 - 3m^2 - 4m + 12)e^{mx} = 0$

$m^2(m-3) - 4(m-3)$

$(m^2-4)(m-3) = 0$

$(m-2)(m+2)(m-3) = 0$

$m = 2, -2, 3$

$m \in N$ के लिए 2 मानों के लिए $y = e^{mx}$ का हल होगा

$m = 2, 3$

* समघात मे परिवर्तन योग्य अवकल समीकरण :-

$\frac{dy}{dx} = \frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}$

$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ $x = X+h$ $y = Y+h$

$dx = dX$ $dy = dY$

$\frac{dY}{dX} = \frac{a_1X+b_1Y+c_1+a_1h+b_1h}{a_2X+b_2Y+c_2+a_2h+b_2h}$

$c_1 + a_1h + b_1h = 0$

$c_2 + a_2h + b_2h = 0$

$\frac{dY}{dX} = \frac{a_1X+b_1Y}{a_2X+b_2Y}$

$y \longrightarrow X$

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$ तो दोनों a_1x+b_1y+c व $a_2x+b_2y+c_2$ के उभयनिष्ठ पद को प्रतिस्थापित करते है

Q $(x-y-2)dx = (2x-2y-3)dy$

$\frac{dy}{dx} = \frac{x-y-2}{2x-2y-3}$ $\left[\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{1}{2}\right.$

$\frac{dy}{dx} = \frac{(x-y)-2}{2(x-y)-3}$

$x-y = t \Rightarrow 1-\frac{dy}{dx} = \frac{dt}{dx}$

$1-\frac{dt}{dx} = \frac{t-2}{2t-3} \Rightarrow \frac{dt}{dx} = 1-\frac{t-2}{2t-3}$

$\frac{dt}{dx} = \frac{2t-3-t+2}{2t-3} \Rightarrow \frac{(2t-3)}{t-1}dt = dx$

$\left(2-\frac{1}{t-1}\right)dt = dx \Rightarrow$

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$2t - \log(t-1) = x + c$

$2(x-y) - \log(x-y-1) = x+c$

$2x - 2y - \log(x-y-1) = x+c$

$\therefore x - 2y = \log(x-y-1) + c$

Q $\dfrac{dy}{dx} = \dfrac{2x-y+1}{x+2y-3}$

$x = X+h \qquad y = Y+k$

$dx = dX \qquad dy = dY$

$\dfrac{dY}{dX} = \dfrac{2X - Y + 1 + 2h - k}{X + 2Y - 3 + h + 2k}$

$1 + 2h - k = 0 \qquad 1 + 2h - k = 0$

$-3 + h + 2k = 0 \qquad -6 + 2h + 4k = 0$

$7 - 5k = 0 \qquad k = 7/5$

$h = 1/5$

$\dfrac{dY}{dX} = \dfrac{2X-Y}{X+2Y}$

$X\dfrac{dv}{dX} = \dfrac{2-v}{1+2v} - v$

$= \dfrac{2-v-v-2v^2}{1+2v}$

$\dfrac{(1+2v)dv}{(2v^2+2v-2)} = -\dfrac{dX}{X}$

$\dfrac{1}{2}\log(v^2+v-1) = -\log X + \log C$

$\dfrac{1}{2}\log\left(\dfrac{Y^2}{X^2} + \dfrac{Y}{X} - 1\right) = -\log X + \log C$

$\dfrac{1}{2}\log\left[\dfrac{(y-7/5)^2}{(x-1/5)^2} + \dfrac{(y-7/5)}{(x-1/5)} - 1\right] = -\log\left(x-\dfrac{1}{5}\right) + \log C$

Q $(3y - 7x + 7)dx + (7y - 3x + 3)dy = 0$

$\dfrac{dy}{dx} = \dfrac{\cancel{7y-3x+3}}{} \quad \dfrac{7x-3y-7}{-3x+7y+3}$

$x = X+h \qquad y = Y+k$

$dx = dX \qquad dy = dY$

$\dfrac{dY}{dX} = \dfrac{7X-3Y}{-3X+7Y}, \quad \left.\begin{array}{l} 7h - 3k - 7 = 0 \\ -3h + 7k + 3 = 0 \end{array}\right\} h = 1, k = 0$

$X\dfrac{dv}{dX} = \dfrac{7-3v}{-3+7v} - v$

$= \dfrac{7-7v^2}{7v-3}$

$\dfrac{(7v-3)dv}{(1-v)(1+v)} = 7\dfrac{dX}{X}$

$\left\{\dfrac{-5}{1+v} + \dfrac{2}{1-v}\right\}dv = 7\dfrac{dX}{X}$

$-5\log(1+v) - 2\log(1-v) = 7\log X + \log C$

$\dfrac{1}{(1+v)^5(1-v)^2} = CX^7$

$v = \dfrac{Y}{X} = \dfrac{y-k}{x-h} = \dfrac{y}{x-1}$

$1 = C(x+y-1)^5(x-y-1)^2$

Q. $(2x+3y-6)\,dy = (6x-2y-7)\,dx$

$\dfrac{dy}{dx} = \dfrac{6x-2y-7}{2x+3y-6}$

$x = X+h \qquad y = Y+k$

$dx = dX \qquad dy = dY$

$\dfrac{dY}{dX} = \dfrac{6X - 2Y - 7 + 6h - 2k}{2X + 3Y - 6 + 2h + 3k} \qquad k = 1, \; h = 3/2$

$$x\frac{dv}{dx} = \frac{6-2v}{2+3v} - v$$

$$x\frac{dv}{dx} = \frac{6-2v-2v-3v^2}{2+3v} = \frac{6-4v-3v^2}{2+3v}$$

$$-\left(\frac{2+3v}{3v^2+4v-6}\right) dv = \frac{dx}{x}$$

$$\frac{1}{2}\log|3v^2+4v-6| = -\log x + \log c$$

$$v = \frac{y}{x} = \frac{y-1}{x-3/2} = \frac{2(y-1)}{(2x-3)}$$

Q $$\frac{dy}{dx} = \frac{(x+y-1)^2}{4(x-2)^2}$$

$x = X+h$, $y = Y+k$

$dx = dX$ $dy = dY$

$$\frac{dY}{dX} = \frac{(X+Y+h+k-1)^2}{4(X+h-2)^2}$$

$$= \frac{(X+Y)^2}{4X^2} \qquad h+k-1=0 \quad k=-1$$

$$h-2=0 \Rightarrow h=2$$

$$X\frac{dV}{dX} = \frac{(1+V)^2}{4} - V$$

$$X\frac{dV}{dX} = (1-V)^2 \Rightarrow \frac{4\,dV}{(1-V)^2} = \frac{dX}{X}$$

$$\frac{4}{(1-V)} = \log X + \log c \qquad \left[\begin{array}{l} X = x-2 \\ Y = y+1 \end{array}\right.$$

$$\frac{4}{(1-V)} = Xc$$

$$\frac{4}{(1-Y/X)} = (x-2)c \Rightarrow \frac{4}{\left(1-\frac{y+1}{x-2}\right)} = (x-2)c$$

$$\frac{4}{(x-2-y-1)} = c \Rightarrow 4 = c(x-y-3) \text{ Ans}$$

रैखिक अवकल समीकरण

$$\frac{dy}{dx} + Py = Q$$

$P, Q \rightarrow x$ के फलन

समाकलन (I.F.)

$$I.F. = e^{\int P\,dx}$$

Solution

$$\boxed{y(I.F.) = \int Q(I.F.)\,dx + C}$$

$$\frac{dx}{dy} + Px = Q$$

$P, Q \rightarrow y$ के फलन

समाकलन गुणांक

$$I.F. = e^{\int P\,dy}$$

Solⁿ

$$\boxed{x(I.F.) = \int Q(I.F.)\,dy + c}$$

Q. $$\frac{dy}{dx} + y\tan x - \sec x = 0$$

Ans $P = \tan x$ $Q = \sec x$

समाकलन गुणांक

$$I.F. = e^{\int \tan x\,dx}$$

$$I.F. = e^{\log\sec x} = \sec x$$

Sol.ⁿ $\Rightarrow$ $y(\sec x) = \int \sec^2 x\,dx + c$

$$\boxed{y(\sec x) = \tan x + c}$$

Q. $$\frac{dy}{dx} + \frac{4x}{(x^2+1)}y = \frac{1}{(x^2+1)^3}$$

Ans $P = \frac{4x}{(x^2+1)}$ $Q = \frac{1}{(x^2+1)^3}$

$$I.F. = e^{\int\frac{4x}{x^2+1}dx} = e^{\frac{4}{2}\log(x^2+1)} = (x^2+1)^2$$

Solⁿ $$y(x^2+1)^2 = \int \frac{(x^2+1)^2}{(x^2+1)^3}\,dx + c$$

$$y(1+x^2)^2 = \int \frac{1}{x^2+1}\,dx + c$$

$$y(1+x^2)^2 = \tan^{-1}x + C$$

I LT

Q. $\frac{dy}{dx} + y = x^{-2}$

$I.F. = e^{\int dx} = e^x$

$y e^x = \int e^x . x^{-2} dx + c$

$y e^x = \cancel{x^2 e^x} - \cancel{\int 2x e^x} + c$

$y e^x = \cancel{x^2 e^x - 2[x e^x - e^x}$

$\underline{y e^x = \int e^x x^{-2} dx + c}$ Ans

Q. $\cancel{(x+y+1)}\ (1+y^2)dx = (\tan^{-1}y - x)dy$

$\frac{dy}{dx} = \frac{\cancel{\tan^{-1}y - x}\ 1+y^2}{\tan^{-1}y - x}$

$\frac{dx}{dy} = \frac{\tan^{-1}y - x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2} - \frac{x}{(1+y^2)}$

$\frac{dx}{dy} + \frac{x}{(1+y^2)} = \frac{\tan^{-1}y}{(1+y^2)}$

$I.F. = e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$

$x e^{\tan^{-1}y} = \int \frac{e^{\tan^{-1}y} . \tan^{-1}y}{1+y^2} dy + c$

$\tan^{-1}y = t \qquad \frac{1}{1+y^2} dy = dt$

$x e^{\tan^{-1}y} = \int e^t . t\ dt + c$

$x e^{\tan^{-1}y} = (te^t - e^t) + c$

$x e^{\tan^{-1}y} = (\tan^{-1}y\ e^{\tan^{-1}y} - e^{\tan^{-1}y}) + c$

Q $(x + 2y^3)\frac{dy}{dx} = y$

Ans $\frac{dx}{dy} = \frac{(x+2y^3)}{y} \qquad \frac{dx}{dy} = \frac{x}{y} + 2y^2$

$\frac{dx}{dy} - \frac{x}{y} = 2y^2$

$I.F = e^{\int -1/y\, dy} = e^{-\log y} = \frac{1}{y}$

solⁿ $x\left(\frac{1}{y}\right) = \int 2y\, dy + c$

$x\left(\frac{1}{y}\right) = \frac{2y^2}{2} + c \Rightarrow \frac{x}{y} = y^2 + c$

Q. $\frac{dy}{dx} + \frac{y}{x} = x^2$

$I.F. = e^{\int \frac{1}{x} dx} = e^{\log x} = x$

$yx = \int x\, x^2 dx + c \Rightarrow yx = \frac{x^4}{4} + c$

Q सभी शांकवो का अवकलन समी ज्ञात करो जिनकी अक्ष निर्देशी अक्षों के सम्पाति है।

Ans $ax^2 + by^2 = c \quad \boxed{\rightarrow x^2 + b/a\, y^2 = c/a}$

$2x + \frac{b}{a} 2yy_1 = 0 \Rightarrow \frac{-2x}{2yy_1} = \frac{b}{a}$

$\frac{yy_1}{x} = A \Rightarrow x(y_1y_1 + yy_2) - yy_1 = 0$

$x\left\{\left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2}\right\} - y.\frac{dy}{dx} = 0$

Q. $x(1-x^2)dy + (2x^2y - y - ax^3)dx = 0$

$\frac{dy}{dx} + \frac{y(2x^2-1)}{x(1-x^2)} = \frac{ax^3}{x(1-x^2)}$

$I.F = \exp.\left\{\int \frac{2x^2-1}{x(1-x^2)} dx\right.$

$I.F = \exp\left\{\frac{-1}{x} + \frac{1}{2(1-x)} - \frac{1}{2(1+x)}\right\}dx$

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$$= \exp\left\{-\log x - \frac{1}{2}\log(1-x) - \frac{1}{2}\log(1+x)\right\}$$

$$= \exp\left\{-\log x - \frac{1}{2}\log(1-x^2)\right\}$$

$$= \frac{1}{x\sqrt{1-x^2}}$$

$$\frac{y}{x\sqrt{1-x^2}} = \int \frac{ax^2}{(1-x^2)} \cdot \frac{1}{x(1-x^2)^{1/2}}\, dx$$

$$\sqrt{1-x^2} = t \Rightarrow \frac{-x\,dx}{\sqrt{1-x^2}} = dt$$

$$= \int -\frac{a\,dt}{t^2}$$

$$= \frac{a}{t} + c$$

$$\frac{y}{x\sqrt{1-x^2}} = \frac{a}{\sqrt{1-x^2}} + c$$

$$\boxed{y = ax + x\sqrt{1-x^2}\,c}$$

Q $x(1+x^2)\frac{dy}{dx} = y(1-x^2) + x^3\log x$

$$\frac{dy}{dx} + \frac{y(x^2-1)}{x(1+x^2)} = \frac{x^3\log x}{x(1+x^2)}$$

$$\frac{x^2-1}{x(1+x^2)} = \frac{A}{x} + \frac{Bx+C}{1+x^2}$$

$$x^2-1 = A(1+x^2) + (Bx+C)x$$

$x=0 \quad A=-1$

$x^2 = 1 = A+B \Rightarrow B = 2$

$x \to 0 = C$

$$= \frac{-1}{x} + \frac{2x}{(1+x^2)}$$

$$= -\log x + \log(1+x^2)$$

$$= \log\left(\frac{1+x^2}{x}\right)$$

$$\text{I.F.} = e^{\log\left(\frac{1+x^2}{x}\right)} = \frac{1+x^2}{x}$$

$$y\left(\frac{1+x^2}{x}\right) = \int x\log x\, dx + c$$

$$y\left(\frac{1+x^2}{x}\right) = \frac{x^2}{2}\log x - \frac{x^2}{4} + c$$

Q. $\frac{dy}{dx} + \frac{y}{(1-x)\sqrt{x}} = 1-\sqrt{x}$

$$\int \frac{1}{(1-x)\sqrt{x}} \qquad \sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}}dx = dt \Rightarrow \frac{1}{\sqrt{x}}dx = 2dt$$

$$\int \frac{2\,dt}{(1-t^2)} = \int \frac{2\,dt}{(1-t^2)} = \frac{dt}{(1-t)} + \frac{dt}{(1+t)}$$

$$= \log(1+t) - \log(1-t)$$

$$= \log\left(\frac{1+t}{1-t}\right) = \log\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)$$

$$\text{I.F.} = \frac{1+\sqrt{x}}{1-\sqrt{x}}$$

$$y\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right) = \int (1+\sqrt{x})\,dx$$

$$= x + \frac{2}{3}x^{3/2} + C$$

Q $x^2 \frac{dy}{dx} + (1-2x)y = x^2$

$\frac{dy}{dx} + \frac{(1-2x)}{x^2} y = 1$

I.F $= e^{\int \left(\frac{1}{x^2} - \frac{2}{x}\right) dx}$

$e^{-\frac{1}{x} - 2\log x} = \frac{e^{-1/x}}{x^2}$

$y\left(\frac{e^{-1/x}}{x^2}\right) = \int \frac{e^{-1/x}}{x^2} \cdot dx$ $\qquad -\frac{1}{x} = t \qquad + \frac{1}{x^2} dx = dt$

$y\left(\frac{e^{-1/x}}{x^2}\right) = \int e^{-t} dt$

$y\left(\frac{e^{-1/x}}{x^2}\right) = -e^{t} + c \Rightarrow \frac{y(e^{-1/x})}{x^2} = e^{-1/x} + c$

Q. $\frac{dy}{dx} ~~- y \cot~~ \sin x \frac{dy}{dx} + 3y = \cos x$

$\frac{dy}{dx} + 3 \operatorname{cosec} x \cdot y = \cot x$

I.F $= e^{\int 3 \operatorname{cosec} x \, dx} = e^{\log(\tan^3 x/2)}$

$= \tan^3 \frac{x}{2} = (\operatorname{cosec} x - \cot x)^3$

$y \cdot \tan^3 \frac{x}{2} = \int \cos x \cdot (\operatorname{cosec} x - \cot x)^3 dx$

$y \cdot \tan^3 \frac{x}{2} = \int \frac{1 - \tan^2 x/2}{2 \tan x/2} \cdot \tan^3 x/2 \, \tan^2 x/2 \, dx$

$= \int \frac{\tan^2 x/2}{2} dx - \frac{1}{2} \int (\tan^2 x/2)(\sec^2 x/2 - 1) dx$

$= \int \frac{\sec^2 x/2 - 1}{2} dx - \frac{1}{2} \int \tan^2 x/2 \, (\sec^2 x/2 - 1) dx$

$= 2 \tan \frac{x}{2} - x - \frac{\tan^3(x/2)}{3} + c$

Q $(x+y+1)\frac{dy}{dx} = 1$

$\frac{dy}{dx} = 1 + x + y = \frac{dx}{dy}$

$\frac{dx}{dy} - x = 1 + y$

I.F $e^{\int -1 dx} = \frac{1}{e^x}$

$\frac{y}{e^x} = \int (y+1) \frac{1}{e^x} dx + C$

$\frac{y}{e^x} \quad \int (y+1) e^{-x} dx + C$

$y e^{-x} = -e^{-x}(y+1) + C$

बरनौली अवकल समीकरण

$\frac{dy}{dx} + Py = Q y^n$

$y^{-n} \frac{dy}{dx} + P y^{1-n} = Q$

$y^{1-n} = t \Rightarrow (1-n) y^{-n} \frac{dy}{dx} = \frac{dt}{dx}$

$\frac{1}{(1-n)} \frac{dt}{dx} + Pt = Q$

$\frac{dt}{dx} + (1-n)Pt = (1-n)Q$

$t(\text{I.F.}) = \int (1-n) Q (\text{I.F.}) dx + C$

Q. $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$

$\frac{dy}{dx} = e^{2x}e^{-y} - e^x e^{y}$

$\frac{dy}{dx} + e^x = e^{2x}e^{-y}$

$e^y \frac{dy}{dx} + e^x e^{-y} = e^{2x}$

$e^{-y} = t \Rightarrow + e^{-y}\frac{dy}{dx} = \frac{dt}{dx}$

$+\frac{dt}{dx} + e^x t = e^{2x}$

I.F. $= e^{\int e^x} = e^{e^x}$

$e^y e^{e^x} = \int e^x e^x e^{e^x} dx$

$e^x = t$

$e^y e^{e^x} = \int t e^t dt$

$e^y e^{e^x} = (te^t - e^t) + c$

$e^y e^{e^x} = e^x e^{e^x} - e^{e^x} + c$

$e^y = (e^x - 1) + ce^{-e^x}$

Q $(y^3x^3 + xy)\, dy - dx = 0$

$\frac{dy}{dx} - \frac{1}{xy(x^2y^2+1)} = 0$

$\frac{dy}{dx} = \frac{1}{xy(x^2y^2+1)}$

$\frac{dx}{dy} = xy(x^2y^2+1)$

$\frac{dx}{dy} - xy = x^3y^3$

$\frac{-1}{x^2} = t \Rightarrow \frac{2}{x^3}\frac{dx}{dy} = \frac{dt}{dy}$

$\frac{1}{2}\frac{dt}{dy} + yt = y^3$

$\frac{dt}{dy} + 2ty = 2y^3$

I.F. $= e^{\int 2y\,dy} = e^{y^2}$

$t.e^{y^2} = \int 2y^3 e^{y^2} dy + c$ $\qquad y^2 = t$

$t.e^{y^2} = \int te^t\, dt + c$

$\left(\frac{-1}{x^2}\right)e^{y^2} = (te^t - e^t) + c$

$\left(\frac{-1}{x^2}\right)e^{y^2} = \left(\cancel{\frac{-1}{x^2}} y^2e^{y^2} - e^{y^2}\right) + c$

$\left(\frac{-1}{x^2}\right) = (y^2-1) + ce^{-y^2}$

Q $\frac{dy}{dx} + \frac{y}{x}\log y = \frac{y}{x^2}(\log y)^2$

$\frac{1}{y(\log y)^2}\frac{dy}{dx} + \frac{1}{x(\log y)} = \frac{1}{x^2}$

$1/(\log y) = t$

$\frac{-1}{y(\log y)^2}\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} - \frac{t}{x} = \frac{1}{x^2}$

I.F. $= e^{\int -1/x\, dx} = 1/x$

$\frac{1}{x\log y} = \int \frac{-1}{x^3} dx + c$

$\frac{1}{x\log y} = +\frac{1}{2x^2} + c$

Q. $\frac{dy}{dx} = \frac{x^2+y^2+1}{2xy}$

$= \frac{x}{2y} + \frac{y^2+1}{2xy}$ या $\frac{x^2+1}{2xy} + \frac{y}{2x}$

$\frac{dy}{dx} - \frac{y}{2x} = \frac{x^2+1}{2xy}$

$y\frac{dy}{dx} - \frac{y^2}{2x} = \frac{x^2+1}{2x}$

$y^2 = t \qquad \frac{1}{2}\frac{dt}{dx} = y\frac{dy}{dt}$

$\frac{1}{2}\frac{dt}{dx} - \frac{t}{2x} = \frac{x^2+1}{2x}$

$\frac{dt}{dx} - \frac{t}{x} = \frac{x^2+1}{2x}$

I.F. $= e^{\int -1/x\, dx} = \frac{1}{x}$

$t \cdot \frac{1}{x} = \int \frac{(x^2+1)}{x^2}\, dx + c$

$\frac{t}{x} = \int \left(1 + \frac{1}{x^2}\right) dx + c$

$\frac{y^2}{x} = \left(x - \frac{1}{x}\right) + c$

$y^2 = (x^2-1) + cx$

Q $2\frac{dy}{dx} - y\sec x = y^3 \tan x$

$\frac{2}{y^3}\frac{dy}{dx} - \frac{1}{y^2}\sec x = \tan x$

$-\frac{1}{y^2} = t \Rightarrow \frac{2}{y^3}\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} + t\sec x = \tan x$

I.F. $= e^{\int \sec x\, dx} = (\sec x + \tan x)$

$t \cdot (\sec x + \tan x) = \int (\sec x + \tan x)\tan x\, dx + c$

$-\frac{1}{y^2}(\sec x + \tan x) = \int (\sec x \tan x + \tan^2 x)\, dx + c$

$= \int (\sec x \tan x + \sec^2 x - 1)\, dx + c$

$-\frac{1}{y^2}(\sec x + \tan x) = \sec x + \tan x - x + c$

$-\frac{1}{y^2} = 1 + \frac{(-x)}{(\tan x + \sec x)}$

Exact diff equation :- यथायत अवकल समीकरण

$M dx + N dy = 0$

$M, N \rightarrow x, y$ के फलन है

Exact है यदि $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$U = \int M\, dx$
y- अचर

$V = \int \underbrace{\left(N - \frac{\partial U}{\partial y}\right)}_{y \text{ का फलन}} dy$

Sol^n $\boxed{U + V = C}$

Q $(x+y)^2 dx - (y^2 - 2xy - x^2)\, dy = 0$

$\frac{\partial M}{\partial y} = 2(x+y) \qquad \frac{\partial N}{\partial x} = -(-2y - 2x)$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$U = \int (x+y)^2 dx = \frac{(x+y)^3}{3}$

$$N - \frac{\partial u}{\partial y} = -y^2 + 2xy + x^2 - \left\{ \frac{3(x+y)^2}{3} \right\}$$

$$N - \frac{\partial u}{\partial y} = -y^2 + 2xy + x^2 - x^2 - y^2 - 2xy$$

$$= -2y^2$$

$$v = \int \left(N - \frac{\partial u}{\partial y}\right) dy = \frac{-2y^3}{3}$$

$$\boxed{\frac{(x+y)^3}{3} - \frac{2y^3}{3} = c}$$

* $\underline{x^2dx} + \underline{2xy\,dx} + \underline{y^2dx} - y^2dy + \underline{2xy\,dy} + \underline{x^2dy} = 0$

$$\frac{x^3}{3} + x^2y + y^2x - \frac{y^3}{3} = c$$

Q $(y^2e^{xy^2} + 4x^3)\,dx + (2xye^{xy^2} - 3y^2)\,dy = 0$

Ans $\frac{\partial M}{\partial y} = 2y\,e^{xy^2} + y^2 \cdot 2xy\,e^{xy^2}$

$$\frac{\partial N}{\partial x} = 2y\,e^{xy^2} + (2xy)\,y^2e^{xy^2}$$

$$u = \int M dx = e^{xy^2} + x^4$$

$$v = \int \{(2xye^{xy^2} - 3y^2) - 2xye^{xy^2}\}dy = -y^3$$

$$e^{xy^2} + x^4 - y^3 = c$$

* $y^2e^{xy^2}dx + 4x^3dx + \underline{2xye^{xy^2}dy} - 3y^2dy$

$$= \frac{y^2e^{xy^2}}{y^2} + x^4 - y^3 = c$$

$$e^{xy^2} + x^4 - y^3 = c$$

Q $y\sin 2x\,dx - (1 + y^2 + \cos^2 x)\,dy = 0$

Ans $\frac{\partial M}{\partial y} = \sin 2x \qquad \frac{\partial N}{\partial x} = -(-2\cos x \sin x)$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$u = \int M dx$$

$$u = \int y \sin 2x\,dx$$

$$u = \frac{-y\cos 2x}{2}$$

$$v = \int \left(N - \frac{\partial u}{\partial y}\right) dy$$

$$v = \int -1 - y^2 - \cos^2 x + \frac{\cos 2x}{2}$$

$$v = \int \left(-1 - y^2 - \frac{1}{2} - \frac{\cos 2x}{2} + \frac{\cos 2x}{2}\right) dy$$

$$v = \left(-\frac{3}{2}y - \frac{y^3}{3}\right)$$

$$u + v = c \Rightarrow \frac{-y\cos 2x}{2} - \frac{3}{2}y - \frac{y^3}{3} = c$$

* $y\sin 2x\,dx - dy - y^2dy - \cancel{\cos^2 x\,dy} \Rightarrow dy/_2 - \frac{\cos 2x}{2}dy = 0$

$$\frac{-y\cos 2x}{2} - y - \frac{y^3}{3} - \frac{y}{2} = c$$

$$\frac{y\cos 2x}{2} + \frac{3y}{2} + \frac{y^3}{3} = c$$

Q. $(1 + e^y)\cos x\,dx + e^y \sin x\,dy = 0$

$\frac{\partial M}{\partial y} = e^y \cos x \qquad \frac{\partial N}{\partial x} = e^y \cos x$

* $\cos x\,dx + e^y\cos x\,dx + e^y \sin x\,dy = 0$

$$\sin x + e^y \sin x = c$$

Q $(1 + e^{x/y})\,dx + e^{x/y}\left(1 - x/y\right)dy = 0$

$\frac{\partial M}{\partial y} = e^{x/y} \cdot \left(\frac{-x}{y^2}\right) \qquad \frac{\partial N}{\partial x} = e^{x/y} \cdot \left(\frac{1}{y}\right) - e^{x/y}\left(\frac{1}{y}\right) \cdot \frac{x}{y} - e^{x/y}\left(\frac{1}{y}\right.$

$1\,dx + e^{x/y}dx + e^{x/y}dy - \frac{x}{y}e^{x/y}dy = 0$

$x + \frac{e^{x/y}}{1/y}\left(\frac{x^2}{2y}\right) = c$

$\left[\begin{array}{l} ye^{x/y} = e^{x/y}dy \\ ye^{x/y} = -ye^{x/y}\left(\frac{x}{y^2}\right)dy \end{array}\right.$

$x + ye^{x/y} = c$

Q $(x^2+y^2+x)\,dx - (2x^2+2y^2-y)\,dy = 0$ Exact ✗

Q $\cos x(\cos x - \sin\alpha\sin y)\,dx + \cos y(\cos y - \sin\alpha\sin x)\,dy = 0$

$\frac{\partial M}{\partial y} = -\cos x\sin\alpha\cos y$

$\frac{\partial N}{\partial x} = -\cos x\sin\alpha\cos y$

$\cos^2 x\,dx - \cos x\sin\alpha\sin y\,dx + \cos^2 y\,dy - \cos y\sin\alpha\sin x\,dy = 0$

$\frac{1+\cos 2x}{2} - \cos x\sin\alpha\sin y\,dx + \frac{1+\cos 2y}{2}dy - \cos y\sin\alpha\sin x\,dy = c$

$\frac{x}{2} + \frac{\sin 2x}{4} - \sin x\sin\alpha\sin y + \frac{1}{2}y + \frac{\sin 2y}{4} = c$

$\frac{x+y}{2} + \frac{\sin 2x}{4} + \frac{\sin 2y}{4} - \sin x\sin\alpha\sin y = 0$

Exam

Q $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$

$e^y dy = (e^x + x^2)\,dx$

$e^y = e^x + \frac{x^3}{3} + c$

Q. $\frac{dy}{dx} + Py = Qy^n$

क्या प्रतिस्थापित करेंगे

Ans $y^{-n}\frac{dy}{dx} + Py^{1-n} = Q$

y^{1-n} को प्रतिस्थापित करेंगे।

Q. $(x+y)(dx-dy) = dx+dy$

$dx - dy = \frac{dx+dy}{x+y}$

$x - y = \log(x+y) + c$

Q $(x+2y^3)\frac{dy}{dx} = y$

$\frac{dx}{dy} = \frac{x+2y^3}{y} \Rightarrow \frac{dx}{dy} = \frac{x}{y} + 2y^2$

$\frac{dx}{dy} - \frac{x}{y} = 2y^2$ I.F. $= e^{\int -\frac{1}{y}dy} = \frac{1}{y}$

$x\cdot\frac{1}{y} = \int 2y^2\cdot\frac{1}{y}\,dy \Rightarrow \frac{x}{y} = y^2 + c$

$x = y^3 + cy$

Q. $x\cdot\log x\,\frac{dy}{dx} + y = 2\log x$

$\frac{dy}{dx} + \frac{y}{x\log x} = \frac{2}{x}$

I.F. $= e^{\int\frac{1}{x\log x}dx} = e^{\log(\log x)} = \log x$

$y\cdot\log x = \int\frac{2}{x}\log x\,dx$

$\left[\begin{array}{l}\log x = t \\ \frac{1}{x}dx = dt\end{array}\right.$

$y\cdot\log x = 2\,\frac{(\log x)^2}{2} + C$

Q. $xp + 2y = pxy$ $\left[p = \frac{dy}{dx}\right]$

$x\frac{dy}{dx} + 2y = \frac{dy}{dx}xy \Rightarrow 2y = \frac{dy}{dx}(xy - x)$

$\frac{2\,dx}{x} = \frac{dy\,(y-1)}{y}$

$2\log x = y - \log y + c$