Past Year JEE Questions

Questions

Quetion: 01

Solution of the differential equation $ydx + (x + x^2y)\,dy = 0$ is

A. $\log y = Cx$

B. $-\dfrac{1}{xy} + \log y = C$

C. $\dfrac{1}{xy} + \log y = C$

D. $-\dfrac{1}{xy} = C$

Solutions

Solution: 01

Explanation

$ydx + (x + x^2y)\,dy = 0$

$\Rightarrow \dfrac{dx}{dy} = -\dfrac{x}{y} - x^2$

$\Rightarrow \dfrac{dx}{dy} + \dfrac{x}{y} = -x^2,$

It is Bernoullis form. Divide by x^2

$x^{-2}\dfrac{dx}{dy} + x^{-1}\left(\dfrac{1}{y}\right) = -1.$

put $x^{-1} = t,\ -x^{-2}\dfrac{dx}{dy} = \dfrac{dt}{dy}$

We get, $-\dfrac{dt}{dy} + t\left(\dfrac{1}{y}\right) = -1$

$\Rightarrow \dfrac{dt}{dy} - \left(\dfrac{1}{y}\right)t = 1$

It is linear in t.

Integrating factor

$= e^{\int -\frac{1}{y}dy} = e^{-\log y} y^{-1}$

$\therefore$ Solution is

$t\left(y^{-1}\right) = \int \left(y^{-1}\right)dy + c$

$\Rightarrow \dfrac{1}{x}\cdot\dfrac{1}{y} = \log y + c$

$\Rightarrow \log y - \dfrac{1}{xy} = c$