# Differential Equations - Class XII

## Past Year JEE Questions

### Questions

### Quetion: 01

Let y = y(x) be the solution of the differential equation

$\sin x \frac{dy}{dx} + y \cos x = 4x, x \in (0, \pi).$

If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to

A. $-\frac{4}{9}\pi^2$

B. $\frac{4}{9\sqrt{3}}\pi^2$

C. $-\frac{8}{9\sqrt{3}}\pi^2$

D. $-\frac{8}{9}\pi^2$

### Solutions

### Solution: 01

#### Explanation

Given,

sin x $\frac{dy}{dx} + y \cos y = 4x$

⇒ $\frac{dy}{dx}$ + y cot x = 4x cosec x

This is a linear differential equation of form,

$\frac{dy}{dx}$ + py = Q

Where p = cot x and Q = 4x cosec x

So, Integrating factor (I. F)

$= e^{\int p dx}$

$= e^{\int \cot dx}$

$= e^{\ln|\sin x|}$

= sin x as $x \in (0, \pi)$

Solution of the differential equation is

y sin x = $\int$ 4x cosecx sinx dx + c

⇒ y sinx = $\int 4x\, dx + c$

$\Rightarrow$ y sinx = $4.\frac{x^2}{2} + c$

$\Rightarrow$ y sinx = $2x^2$ + c . . . . . (1)

Given that, $y\left(\frac{\pi}{2}\right) = 0$

$\therefore$ x = $y\left(\frac{\pi}{2}\right) = 0$ and y = 0

Put this x = $\frac{\pi}{2}$ and y = 0 at equation (1)

0.1 = 2. $\left(\frac{\pi}{2}\right)^2$ + c

$\Rightarrow$ c = $-\frac{\pi^2}{2}$

So, differential equation is

y sin x = $2x^2 - \frac{\pi^2}{2}$ . . . . . (2)

Now we have to find y $\left(\frac{\pi}{6}\right)$ .

So, put x = $\frac{\pi}{6}$ at equation (2)

y . sin $\frac{\pi}{6}$ = 2 $\left(\frac{\pi}{6}\right)^2 - \frac{\pi^2}{2}$

$\Rightarrow$ $y.\frac{1}{2} = 2.\frac{\pi^2}{36} - \frac{\pi^2}{2}$

$\Rightarrow$ $\frac{y}{2} = \frac{\pi^2}{18} - \frac{\pi^2}{2}$

$\Rightarrow$ $\frac{y}{2} = \frac{\pi^2 - 9\pi^2}{18}$

$\Rightarrow$ $y = -\frac{8\pi^2}{9}$