

Differential Equations - Class XII

Past Year JEE Questions

Questions

Question: 01

Consider the differential equation, $y^2 dx + (x - \frac{1}{y}) dy = 0$, If value of y is 1 when $x = 1$, then the value of x for which $y = 2$, is :

- A. $\frac{3}{2} - \frac{1}{\sqrt{e}}$
- B. $\frac{1}{2} + \frac{1}{\sqrt{e}}$
- C. $\frac{5}{2} + \frac{1}{\sqrt{e}}$
- D. $\frac{3}{2} - \sqrt{e}$

Solutions

Solution: 01

Explanation

$$y^2 dx + (x - \frac{1}{y}) dy = 0$$

$$\Rightarrow \frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$$

Integrating factor (I.F) = $e^{-\frac{1}{y}}$

Now $x \cdot e^{-\frac{1}{y}} = \int e^{-\frac{1}{y}} \frac{1}{y^3} dy$ by putting $-\frac{1}{y} = t$

$$x \cdot e^t = \int e^t (-t) dt$$

$$\Rightarrow x e^t = -(t \cdot e^t - e^t) + c$$

$$\Rightarrow x e^{-\frac{1}{y}} = e^{-\frac{1}{y}} (1 + \frac{1}{y}) + c$$

$$\Rightarrow x = 1 + \frac{1}{y} + c \cdot e^{\frac{1}{y}}$$

It passes through (1, 1)

$$\therefore c = -\frac{1}{e}$$

Equation of the curve is

$$x = 1 + \frac{1}{y} - e^{\frac{1}{y}-1} \text{ It passes through } (k, 2)$$

$$\therefore k = 1 + \frac{1}{2} - e^{\frac{1}{2}-1} = \frac{3}{2} - \frac{1}{\sqrt{e}}$$