58. The solution of $x\frac{dy}{dx}+y=e^x$ is

(a) $y=\frac{e^x}{x}+\frac{k}{x}$ (b) $y=xe^x+cx$ (c) $y=xe^x+k$ (d) $x=\frac{e^y}{y}+\frac{k}{y}$

Sol. **(a)** We have, $x\frac{dy}{dx}+y=e^x$

$$\Rightarrow \quad \frac{dy}{dx}+\frac{y}{x}=\frac{e^x}{x}$$

This is a linear differential equation.

On comparing it with $\frac{dy}{dx}+Py=Q$, we get

$$P=\frac{1}{x} \text{ and } Q=\frac{e^x}{x}$$

$$\therefore \quad \text{I.F.}=e^{\int\frac{1}{x}dx}=e^{(\log x)}=x$$

So, the general solution is:

$$y\cdot x=\int\frac{e^x}{x}x\,dx$$

$$\Rightarrow \quad y\cdot x=\int e^x\,dx$$

$$\Rightarrow \quad y\cdot x=e^x+k$$

$$\Rightarrow \quad y=\frac{e^x}{x}+\frac{k}{x}$$

Basic textbook example of LDEs. And when you look at other problems related to these concepts. They have the same procedure as this one.