

Exemplar Problems

55. The integrating factor of the differential equation  $\frac{dy}{dx} + y = \frac{1+y}{x}$  is

(a)  $\frac{x}{e^x}$

(b)  $\frac{e^x}{x}$

(c)  $xe^x$

(d)  $e^x$

**Sol. (b)** We have,  $\frac{dy}{dx} + y = \frac{1+y}{x}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x} + \frac{y(1-x)}{x}$$

$$\Rightarrow \frac{dy}{dx} - \left(\frac{1-x}{x}\right)y = \frac{1}{x}$$

This is a linear differential equation.

On comparing it with  $\frac{dy}{dx} + Py = Q$ , we get

$$P = \frac{-(1-x)}{x}, Q = \frac{1}{x}$$

$$\text{I.F.} = \int P dx = e^{-\int \frac{1-x}{x} dx}$$

Being able to recognize that it is a LDE, and its right form. That is the main task here.