21. Solve the differential equation dy = cos x (2 – y cosec x) dx given that y=2 when x = π /2.

Sol. We have

$$dy = \cos x\,(2 - y \operatorname{cosec} x)dx$$

$$\Rightarrow \quad \frac{dy}{dx} = 2\cos x - y \operatorname{cosec} x \cdot \cos x$$

$$\Rightarrow \quad \frac{dy}{dx} + y\cot x = 2\cos x$$

This is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

$$P = \cot x,\ Q = 2\cos x$$

$$\text{I.F.} = e^{\int P dx} = e^{\int \cot x\, dx} = e^{\log \sin x} = \sin x$$

Thus, the general solution is:

$$y \cdot \sin x = \int 2\cos x \cdot \sin x\, dx + C$$

$$\Rightarrow \quad y \cdot \sin x = \int \sin 2x\, dx + C$$

$$\Rightarrow \quad y \cdot \sin x = -\frac{\cos 2x}{2} + C$$

Given that when $x = \frac{\pi}{2}$ and $y = 2$

$$\Rightarrow \quad 2 \cdot \sin\frac{\pi}{2} = -\frac{\cos \pi}{2} + C$$

$$\Rightarrow \quad 2 = \frac{1}{2} + C$$

$$\Rightarrow \quad C = \frac{3}{2}$$

On substituting the value of C in Eq. (i), we get

$$y \sin x = -\frac{1}{2}\cos 2x + \frac{3}{2}$$