17. Find the general solution of the differential equation $(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0.$

Sol. Given, differential equation is

$$(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$$

$$\Rightarrow \quad (1 + y^2)\frac{dx}{dy} + x - e^{\tan^{-1} y} = 0$$

$$\Rightarrow \quad \frac{dx}{dy} + \frac{x}{1 + y^2} = \frac{e^{\tan^{-1} y}}{1 + y^2}$$

This is a linear differential equation.

On comparing it with $\frac{dx}{dy} + Px = Q$, we get

$$P = \frac{1}{1 + y^2}, Q = \frac{e^{\tan^{-1} y}}{1 + y^2}$$

$$\text{I.F.} = e^{\int P dx} = e^{\int \frac{1}{1+y^2} dx} = e^{\tan^{-1} y}$$

So, the general solution is:

$$x \cdot e^{\tan^{-1} y} = \int \frac{e^{\tan^{-1} y}}{1 + y^2} \cdot e^{\tan^{-1} y} dx + C$$

Put $\quad e^{\tan^{-1} y} = t$

$$\Rightarrow \quad \frac{e^{\tan^{-1} y}}{1 + y^2} dy = dt$$

$$\therefore \quad x \cdot e^{\tan^{-1} y} = \int t \, dt + C$$

$$\Rightarrow \quad xe^{\tan^{-1} y} = \frac{t^2}{2} + C$$

$$\Rightarrow \quad xe^{\tan^{-1} y} = \frac{e^{2\tan^{-1} y}}{2} + C$$