

15. Find the equation of a curve passing through origin and satisfying the differential equation $(1 + x^2) \frac{dy}{dx} + 2xy = 4x^2$.

Sol. We have, $(1 + x^2) \frac{dy}{dx} + 2xy = 4x^2$

$$\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2} \cdot y = \frac{4x^2}{1+x^2}$$

This is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

$$P = \frac{2x}{1+x^2}, Q = \frac{4x^2}{1+x^2}$$

$$\therefore \text{I.F.} = e^{\int P dx} = e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = (1+x^2)$$

So, the general solution is:

$$y \cdot (1+x^2) = \int \frac{4x^2}{1+x^2} (1+x^2) dx + C$$

$$\Rightarrow y \cdot (1+x^2) = \int 4x^2 dx + C$$

$$\Rightarrow y \cdot (1+x^2) = \frac{4x^3}{3} + C \quad \text{(i)}$$

Since, the curve passes through origin, then putting $x = 0$ and $y = 0$ in Eq. (i), we get $C = 0$

$$\text{So, the required equation of curve is: } y = \frac{4x^3}{3(1+x^2)}$$