5) If roots of the equation $ax^2+bx+c=0$ are α and β, find the equation whose roots are.

(i) $\frac{1}{\alpha}, \frac{1}{\beta}$ (ii) $-\alpha, -\beta$ (iii) $\frac{1-\alpha}{1+\alpha}, \frac{1-\beta}{1+\beta}$

Solution: Here in all cases functions of α and β are symmetric.

(i) Let $\frac{1}{\alpha} = y \Rightarrow \alpha = \frac{1}{y}$

Now α is a root of the equation $ax^2+bx+c=0$

$\Rightarrow a\alpha^2+b\alpha+c=0$

$\Rightarrow \frac{a}{y^2}+\frac{b}{y}+c=0$

$\Rightarrow cy^2+by+a=0.$

Hence, the required equation is $cx^2+bx+a=0$. We get ~~the~~ same equation if we start with $1/\beta$.

(ii) Let $-\alpha = y \Rightarrow \alpha = -y$

Now α is root of the equation $ax^2+bx+c=0$

$\Rightarrow a\alpha^2+b\alpha+c=0$

$\Rightarrow a(-y)^2+b(-y)+c=0$

Hence the required equation is $ax^2-bx+c=0$

(iii) Let $\frac{1-\alpha}{1+\alpha} = y \Rightarrow \alpha = \frac{1-y}{1+y}$

Now α is root of the equation $ax^2+bx+c=0$

$\Rightarrow a\alpha^2+b\alpha+c=0$

$\Rightarrow a\left(\frac{1-y}{1+y}\right)^2 + b\left(\frac{1-y}{1+y}\right) + c = 0$

Hence required equation is $a(1-x)^2+b(1-x^2)+c(1+x)^2=0.$