4) If $f(x) = ax^2 + bx + c$, $g(x) = -ax^2 + bx + c$, where $ac \neq 0$, then prove that $f(x)\,g(x) = 0$ has at least two real roots.

Solution: Let D_1 and D_2 be discriminants of $ax^2 + bx + c = 0$ and $-ax^2 + bx + c = 0$

Then, $D_1 = b^2 - 4ac$, $D_2 = b^2 + 4ac$

Now,

$ac \neq 0 \Rightarrow$ either $ac > 0$ or $ac < 0$.

If $ac > 0$, then $D_2 > 0$. Therefore, roots of $-ax^2 + bx + c = 0$ are real.

If $ac < 0$, then $D_1 > 0$. Therefore, roots of $ax^2 + bx + c = 0$ are real.

Thus, $f(x)\,g(x)$ has atleast two real roots