

2) If the roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30^\circ$ and $\tan 15^\circ$, respectively, then find the value of $2 + q - p$.

Solution: The equation $x^2 + px + q = 0$ has roots $\tan 30^\circ$ and $\tan 15^\circ$.

Therefore,

$$\tan 30^\circ + \tan 15^\circ = -p \quad \text{--- (1)}$$

$$\tan 30^\circ \cdot \tan 15^\circ = q \quad \text{--- (2)}$$

Now, $\tan 45^\circ = \tan (30^\circ + 15^\circ)$

$$\rightarrow 1 = \frac{\tan 30^\circ + \tan 15^\circ}{1 - \tan 30^\circ \tan 15^\circ}$$

$$\rightarrow 1 = \frac{-p}{1-q} \quad [\text{Using (1) and (2)}]$$

$$\rightarrow 1 - q = -p \Rightarrow q - p = 1.$$

$$\rightarrow 2 + q - p = 1 + 2 = \boxed{3}$$