5.) Prove that the roots of the equation $(a^4+b^4)x^2+4abcdx+(c^4+d^4)=0$ cannot be different, if real.

Solution: The discriminant of the given equation is:

$$D = 16a^2b^2c^2d^2 - 4(a^4+b^4)(c^4+d^4)$$

$$= -4\left[(a^4+b^4)(c^4+d^4) - 4a^2b^2c^2d^2\right]$$

$$= -4\left[a^4c^4 + a^4d^4 + b^4c^4 + b^4d^4 - 4a^2b^2c^2d^2\right]$$

$$= -4\left[(a^4c^4 + b^4d^4 - 2a^2b^2c^2d^2) + (a^4d^4 + b^4c^4 - 2a^2b^2c^2d^2)\right]$$

$$= -4\left[(a^2c^2 - b^2d^2)^2 + (a^2d^2 - b^2c^2)^2\right] \quad \text{——— (1)}$$

Since roots of the given equation are real, therefore $D \geq 0$.

$$\Rightarrow -4\left[(a^2c^2 - b^2d^2)^2 + (a^2d^2 - b^2c^2)^2\right] \geq 0$$

$$\Rightarrow (a^2c^2 - b^2d^2)^2 + (a^2d^2 - b^2c^2)^2 \leq 0$$

$$\Rightarrow (a^2c^2 - b^2d^2)^2 + (a^2d^2 - b^2c^2)^2 = 0 \quad \text{——— (2)}$$

(Since sum of two positive quantities cannot be negative)

From (1) and (2), we get $D=0$. Hence, the roots of the given quadratic equation are not different, if real.