4) If the roots of the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ are equal, show that $2/b = \frac{1}{a} + \frac{1}{c}$.

Solution: Since the roots of the given equations are equal, therefore its discriminant is zero,

$$b^2(c-a)^2 - 4a(b-c)c(a-b) = 0$$

$$\Rightarrow b^2(c^2 + a^2 - 2ac) - 4ac(ba - ca - b^2 + bc) = 0$$

$$\Rightarrow a^2b^2 + b^2c^2 + 4a^2c^2 + 2b^2ac - 4a^2bc - 4abc^2 = 0$$

$$\Rightarrow (ab + bc - 2ac)^2 = 0$$

$$\Rightarrow ab + bc - 2ac = 0$$

$$\Rightarrow ab + bc = 2ac$$

$$\Rightarrow \frac{1}{c} + \frac{1}{a} = \frac{2}{b} \qquad [\text{Dividing both sides by } abc]$$

$$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$