Questions

Quetion: 01

The derivative of

$\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with

respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x = \frac{1}{2}$ is :

A. $\frac{2\sqrt{3}}{3}$

B. $\frac{2\sqrt{3}}{5}$

C. $\frac{\sqrt{3}}{10}$

D. $\frac{\sqrt{3}}{12}$

Solutions

Solution: 01

Explanation

Let $f = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$f = \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$

$\Rightarrow f = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right) = \frac{\theta}{2}$

$\Rightarrow f = \frac{\tan^{-1}x}{2}$

$\therefore \frac{df}{dx} = \frac{1}{2(1+x^2)}\ldots(1)$

Let $g = \tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$

Put $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$

$\Rightarrow g = \tan^{-1}\left(\frac{2\sin\theta\cos\theta}{1-2\sin^2\theta}\right)$

$\Rightarrow g = \tan^{-1}(\tan 2\theta) = 2\theta$

$\Rightarrow g = 2\sin^{-1}x$

$\Rightarrow \frac{dg}{dx} = \frac{2}{\sqrt{1-x^2}}\ldots(2)$

Using (i) and (ii),

$$\therefore \frac{df}{dg} = \frac{1}{2(1+x^2)} \frac{\sqrt{1-x^2}}{2}$$

At $x = \frac{1}{2}$, $\left(\frac{df}{dg}\right)_{x=\frac{1}{2}} = \frac{\sqrt{3}}{10}$