42. $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right), \frac{1}{\sqrt{3}} < \frac{x}{a} < \frac{1}{\sqrt{3}}$

Sol. Let $y = \tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right) = \tan^{-1}\left(\frac{3\frac{x}{a} - \left(\frac{x}{a}\right)^3}{1 - 3\left(\frac{x}{a}\right)^2}\right)$

Put $x = a\tan\theta \Rightarrow \theta = \tan^{-1}\frac{x}{a}$

$\therefore \quad y = \tan^{-1}\left[\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right] = \tan^{-1}(\tan 3\theta) = 3\theta = 3\tan^{-1}\frac{x}{a}$

$\therefore \quad \frac{dy}{dx} = 3\frac{d}{dx}\tan^{-1}\frac{x}{a}$

$= 3\left[\frac{1}{1 + \frac{x^2}{a^2}}\right] \cdot \frac{d}{dx}\left(\frac{x}{a}\right) = \frac{3a^2}{a^2 + x^2} \cdot \frac{1}{a} = \frac{3a}{a^2 + x^2}$