Question 5. If in a triangle $a\cos^2 \frac{C}{2} + c\cos^2 \frac{A}{2} = \frac{3b}{2}$, then find the relation between the sides of the triangle.

Solution.

$$a\cos^2 \frac{C}{2} + c\cos^2 \frac{A}{2} = \frac{3b}{2}$$

$\Rightarrow \quad a(1 + \cos\ C) + c(1 + \cos\ A) = 3b$

$\Rightarrow \quad a + c + (a\cos\ C + c\cos\ A) = 3b$

$\Rightarrow \quad a + c + b = 3b$ [by the projection formula]

$\quad\quad \Rightarrow a + c = 2b$

$\Rightarrow \quad a, b, c$ are in A.P.