

Question 4. Prove that  $a(b \cos C - c \cos B) = b^2 - c^2$ .

Solution.  $a(b \cos C - c \cos B) = (b \cos C + c \cos B)(b \cos C - c \cos B)$

$$\begin{aligned} &= b^2 \cos^2 C - c^2 \cos^2 B \\ &= b^2(1 - \sin^2 C) - c^2(1 - \sin^2 B) \\ &= b^2 - c^2 - (b^2 \sin^2 C - c^2 \sin^2 B) \\ &= b^2 - c^2 \text{ [ as by the sine rule } b \sin C = c \sin B \text{ ]} \end{aligned}$$