

Question 3. Triangle  $ABC$  has  $BC = 1$  and  $AC = 2$ . Find the maximum possible value of angle  $A$

Solution.

Using cosine rule, we have

$$\begin{aligned}\cos \theta &= \frac{x^2 + 4 - 1}{4x} \\ &= \frac{x^2 + 3}{4x} \\ &= \frac{1}{4} \left[ x + \frac{3}{x} \right] \\ &= \frac{1}{4} \left[ \left( \sqrt{x} - \sqrt{\frac{3}{x}} \right)^2 + 2\sqrt{3} \right]\end{aligned}$$

Hence,  $\cos \theta$  is minimum if  $x = \sqrt{3}$ .

Therefore, the minimum value of  $\cos \theta = 2 \times \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$ , and the maximum value of  $\theta = \frac{\pi}{6}$