

Question 1. Let  $f, g$  and  $h$  be the lengths of the perpendiculars from the circumcentre of  $\triangle ABC$  on the sides  $a, b$  and  $c$ , respectively, then prove that  $\frac{a}{f} + \frac{b}{g} + \frac{c}{h} = \frac{1}{4} \frac{abc}{fgh}$ .

Solution. Distance of circumcentre from O to side  $BC$  is  $R \cos A = f$ .

Similarly,  $g = R \cos B, h = R \cos C$

$$\Rightarrow \frac{a}{f} + \frac{b}{g} + \frac{c}{h} = \frac{2R \sin A}{R \cos A} + \frac{2R \sin B}{R \cos B} + \frac{2R \sin C}{R \cos C} = 2(\tan A + \tan B + \tan C)$$

Also,  $\frac{a}{f} \frac{b}{g} \frac{c}{h} = 8 \tan A \tan B \tan C$

But in triangle,  $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

$$\Rightarrow \frac{a}{f} + \frac{b}{g} + \frac{c}{h} = \frac{1}{4} \frac{abc}{fgh}$$