Question 4. Let a, b and c be the three sides of a triangle, then prove that the equation $b^2x^2 + \left(b^2 + c^2 - a^2\right)x + c^2 = 0$ has imaginary roots.

Solution.

$$\begin{aligned} & b^2x^2 + \left(b^2 + c^2 - a^2\right)x + c^2 = 0 \\ & \text{Let } f(x) = b^2x^2 + (2bc\cos A)x + c^2 = 0 \end{aligned}$$

Also in $\triangle ABC$, where $A \in (0, \pi)$ in a triangle, we find $\cos A \in (-1, 1)$

$$\begin{aligned} & \Rightarrow 2bc\cos A \in (-2bc, 2bc) \\ & \Rightarrow D = (2bc\cos A)^2 - 4b^2c^2 = 4b^2c^2\left(\cos^2 A - 1\right) < 0. \end{aligned}$$

Hence, the roots are imaginary.