Question 2. Prove that

$$\frac{a^2 \sin(B-C)}{\sin B+\sin C}+\frac{b^2 \sin(C-A)}{\sin C+\sin A}+\frac{c^2 \sin(A-B)}{\sin A+\sin B}=0\cdot$$

Solution.

$$\begin{aligned}\frac{a^2 \sin(B-C)}{\sin B+\sin C} &= \frac{4R^2 \sin^2 A \sin(B-C)}{\sin B+\sin C}\\ &= \frac{4R^2 \sin A \sin(B+C) \sin(B-C)}{\sin B+\sin C}\\ &= \frac{4R^2 \sin A\left(\sin^2 B-\sin^2 C\right)}{\sin B \cdot+\sin C}\\ &= 4R^2 \sin A(\sin B-\sin C)\end{aligned}$$

Similarly,

$$\frac{b^2 \sin(C-A)}{\sin C+\sin A}=4R^2 \sin B(\sin C-\sin A)$$

and

$$\frac{c^2 \sin(A-B)}{\sin A+\sin B}=4R^2 \sin C(\sin A-\sin B)$$

Adding, we get

$$\frac{a^2 \sin(B-C)}{\sin B+\sin C}+\frac{b^2 \sin(C-A)}{\sin C+\sin A}+\frac{c^2 \sin(A-B)}{\sin A+\sin B}=0$$