Question1. In any triangle, if $\frac{a^2-b^2}{a^2+b^2} = \frac{\sin(A-B)}{\sin(A+B)}$, then prove that the triangle is either right angled or isosceles. .
Solution. $\frac{a^2-b^2}{a^2+b^2} = \frac{\sin(A-B)}{\sin(A+B)}$
We know from Sine Rule that:-

$$\Rightarrow \frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$

Therefore, we can write that:-

$$\Rightarrow a = 2R * SinA, b = 2R * SinB$$

Therefore:-

$$\Rightarrow \frac{4R^2 \sin^2 A - 4R^2 \sin^2 B}{4R^2 \sin^2 A + 4R^2 \sin^2 B} = \frac{\sin(A-B)}{\sin(A+B)}$$
$$\Rightarrow \frac{\sin(A+B)\sin(A-B)}{\sin^2 A + \sin^2 B} = \frac{\sin(A-B)}{\sin(A+B)}$$

$$\Rightarrow \sin(A-B) = 0 \text{ or } \frac{\sin(\pi - C)}{\sin^2 A + \sin^2 B} = \frac{1}{\sin(\pi - C)}$$
$$\Rightarrow \quad A = B \text{ or } \sin^2 C = \sin^2 A + \sin^2 B$$

$\Rightarrow A = B$ or $c^2 = a^2 + b^2$ [from the sine rule]
Therefore, the triangle is isosceles or right angled.