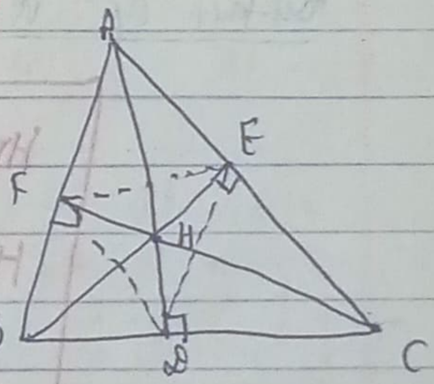
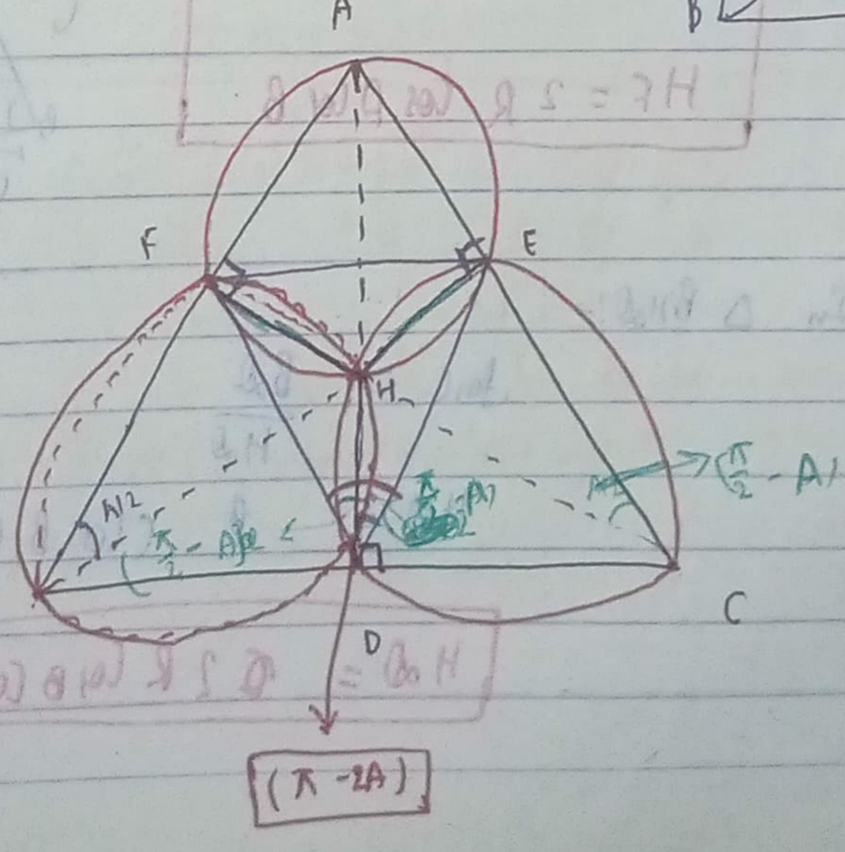
Orthocentre

Pt. of intersection of altitudes.



ΔDEF, triangle formed by feet of $\perp$, is called pedal triangle.



Dimension of Pedal - Triangle:-

Sides:

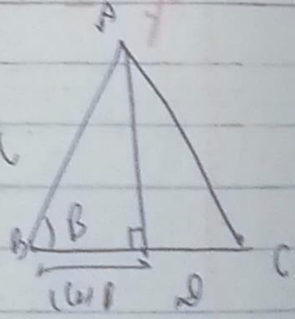
$$\boxed{\begin{aligned} EF &= a\cos A \\ DF &= b\cos B \\ DE &= c\cos C \end{aligned}}$$

Angles:

$$\boxed{\begin{aligned} \angle FDE &= \pi - 2A \\ \angle DEF &= \pi - 2B \\ \angle EFD &= \pi - 2C \end{aligned}}$$

Distance of vertices of ΔDEF from Orthocentre:

$$\boxed{\begin{aligned} HD &= 2R\cos B\cos C \\ HE &= 2R\cos A\cos C \\ HF &= 2R\cos A\cos B \end{aligned}}$$



Proof:

In ΔBHD:-

$$\tan C = \frac{BD}{HD}$$

$$HD = c\cos B\cot C$$

$$\boxed{HD = 2R\cos B\cos C}$$

~~Note~~ :-

Distance of ~~vertex~~ ~~of~~ orthocentre ~~of~~ from vertex of ΔABC :-

$$\boxed{\begin{aligned} HB &= 2R\cos B \\ HA &= 2R\cos A \\ HC &= 2R\cos C \end{aligned}}$$

* M.I ~~Note~~ :- *

Co-ordinates of 'H' are obtained by solving eq^n of any two altitudes.

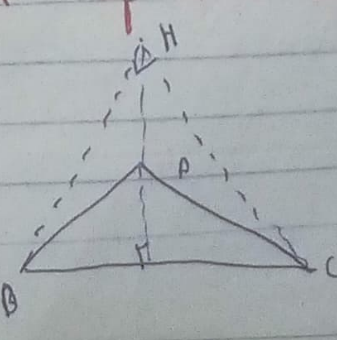
Formula for H :- Prefer to find H :-

$$\boxed{H \equiv \left(\frac{x_1 \tan A + x_2 \tan B + x_3 \tan C}{\tan A + \tan B + \tan C}, \frac{\sum x_1 \tan A}{\sum \tan A} \right)}$$

Note :

* H always lies inside the Δ, for acute triangle.

* H always lies outside the Δ & on the side of obtuse angle, in case of obtuse triangle.



If 'H' is orthocentre of $\triangle ABC$, then

Orthocentre of $\triangle AHB \longrightarrow C$

" " $\triangle AHC \longrightarrow B$

" " $\triangle BHC \longrightarrow A$

* 'H' always lies on the right vertex of $\triangle ABC$

* NB In any triangle, H, G, O are always collinear & 'G' divides HO in the ratio 2:1

i.e. $HG : GO = 2:1$

It is another method to find 'H'.

$H \longrightarrow G \longrightarrow O$

* In case of isosceles triangle, H, G, O & I are collinear.