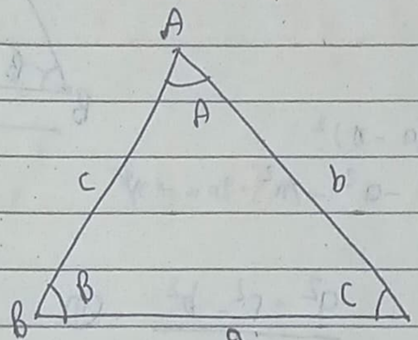
26/02/20

Sol^n of Triangle

$BC = a$

$AC = b$

$AB = c$



$\angle BAC = A$

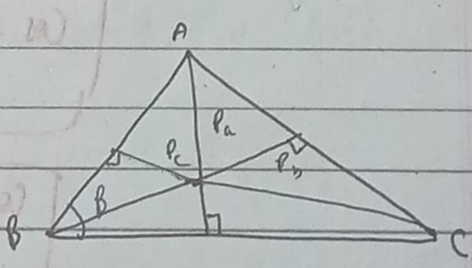
$\angle ABC = B$

$\angle ACB = C$

Perimeter $= (2s) = a + b + c$

Semi-perimeter $= s = \dfrac{a+b+c}{2}$

Sine Rule: →



$\sin B = \dfrac{p_a}{c}$

$\sin C = \dfrac{p_a}{b}$

$\therefore \quad p_a = b\sin C = c\sin B$

$$\frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{— (I)}$$

Similarly $\quad p_B = c\sin A = a\sin C$

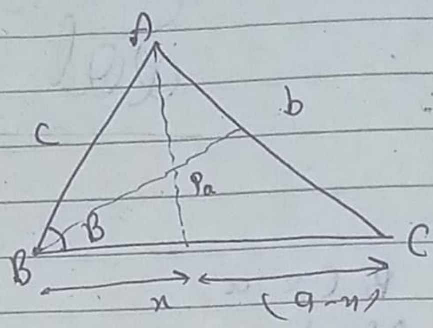
$$\frac{c}{\sin C} = \frac{a}{\sin A} \quad \text{— (II)}$$

$$\therefore \quad \boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R} \rightarrow \text{Circumradius}$$

REDMI NOTE 9

Cosine Rule:→

$$\cos B = \frac{x}{c} \quad \text{—(i)}$$



$$p_a^2 = c^2 - x^2$$
$$= b^2 - (a-x)^2$$
$$\Rightarrow c^2 = b^2 - a^2 - \cancel{x^2} + 2ax + \cancel{x^2}$$

$$x = \frac{a^2 + c^2 - b^2}{2a} \quad \text{—(ii)}$$

∴

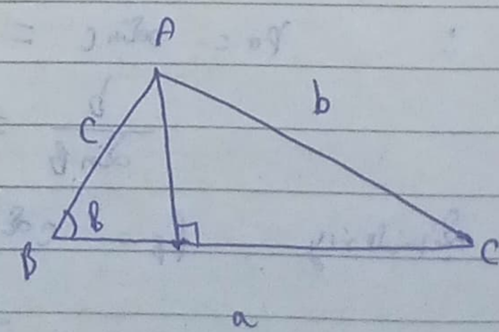
$$\boxed{\cos B = \frac{c^2 + a^2 - b^2}{2ac}}$$

Similarly:→

$$\boxed{\cos A = \frac{b^2 + c^2 - a^2}{2bc}}$$

$$\boxed{\cos C = \frac{a^2 + b^2 - c^2}{2ab}}$$

Projection Rule:→



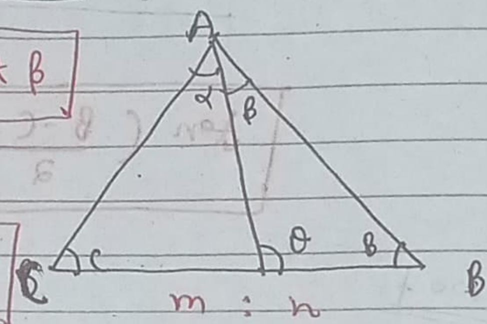
$$\boxed{\begin{aligned} a &= c\cos B + b\cos C \\ b &= a\cos C + c\cos A \\ c &= a\cos B + b\cos A \end{aligned}}$$

m-n Rule

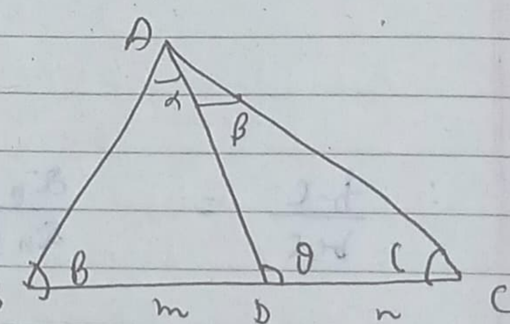
$$\boxed{(m+n)\cot\theta = m\cot\alpha - n\cot\beta}$$

or

$$\boxed{(m+n)\cot\theta = n\cot C - m\cot B}$$



Proof:



By exterior angle theorem:

$$\boxed{\theta = \alpha + B}$$

& $\theta + C + \beta = \pi$, $\alpha + B + \pi - \theta = \pi$

$$\boxed{\alpha + B = \theta} \qquad \alpha + \beta = A$$

In Δ ABD:

$$\frac{BD}{\sin\alpha} = \frac{AD}{\sin B} \quad \text{— (i)}$$

In Δ ACD:

$$\frac{CD}{\sin\beta} = \frac{AD}{\sin C} \quad \text{— (ii)}$$

Now

$$\frac{m}{n} = \frac{BD}{DC} = \frac{\sin\alpha}{\sin\beta} \cdot \frac{\sin C}{\sin B} \quad \text{— (iii)}$$

$$\frac{m}{n} = \frac{\sin\alpha \, \sin[\pi - (\theta+\beta)]}{\sin\beta \, \sin(\theta-\alpha)}$$

$$= \frac{\sin\alpha \, [\sin\theta \cos\beta + \cos\theta \sin\beta]}{\sin\beta \, [\sin\theta \cos\alpha - \cos\theta \sin\alpha]}$$

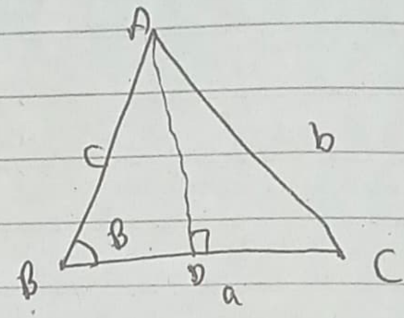
$$= \frac{\sin\alpha \cdot \cancel{\sin\beta \sin\theta} \, [\cot\beta + \cot\theta]}{\sin\alpha \, \cancel{\sin\beta \sin\theta} \, [\cot\alpha - \cot\theta]}$$

$$m\cot\alpha - m\cot\theta = n\cot\beta + n\cot\theta$$

$$\Rightarrow \quad m\cot\alpha - n\cot\beta = (m+n)\cot\theta$$

$$\Rightarrow \quad \boxed{(m+n)\cot\theta = m\cot\alpha - n\cot\beta}$$

Area of Triangle



$\therefore \sin B = \frac{AD}{c}$

$\therefore AD = c \sin B$ — (1)

$\therefore \text{Area} = \frac{1}{2} \times BC \times AD$

$$\boxed{\text{Area} = \frac{1}{2} ac \sin B = \frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A}$$

$$= \frac{1}{2} ac \sqrt{\frac{(s-a)(s-c)}{ac}} \times 2\sqrt{\frac{s(s-b)}{ac}}$$

Heron's Formula

$$\boxed{\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}}$$