EXERCISE 5.2

Find the modulus and the arguments of each of the complex numbers in Exercises 1 to 2.

1. $z = -1 - i\sqrt{3}$ **2.** $z = -\sqrt{3} + i$

Convert each of the complex numbers given in Exercises 3 to 8 in the polar form:

3. $1 - i$ **4.** $-1 + i$ **5.** $-1 - i$

6. -3 **7.** $\sqrt{3} + i$ **8.** i

5.6 Quadratic Equations

We are already familiar with the quadratic equations and have solved them in the set of real numbers in the cases where discriminant is non-negative, i.e., ≥ 0,

Let us consider the following quadratic equation:

$ax^2 + bx + c = 0$ with real coefficients a, b, c and $a \neq 0$.

Also, let us assume that the $b^2 - 4ac < 0$.

Now, we know that we can find the square root of negative real numbers in the set of complex numbers. Therefore, the solutions to the above equation are available in the set of complex numbers which are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{4ac - b^2}\, i}{2a}$$

Note At this point of time, some would be interested to know as to how many roots does an equation have? In this regard, the following theorem known as the *Fundamental theorem of Algebra* is stated below (without proof).

"A polynomial equation has at least one root."

As a consequence of this theorem, the following result, which is of immense importance, is arrived at:

"A polynomial equation of degree n has n roots."

Example 9 Solve $x^2 + 2 = 0$

Solution We have, $x^2 + 2 = 0$

or $x^2 = -2$ i.e., $x = \pm\sqrt{-2} = \pm\sqrt{2}\, i$

Example 10 Solve $x^2 + x + 1 = 0$

Solution Here, $b^2 - 4ac = 1^2 - 4 \times 1 \times 1 = 1 - 4 = -3$

Therefore, the solutions are given by $x = \dfrac{-1 \pm \sqrt{-3}}{2 \times 1} = \dfrac{-1 \pm \sqrt{3}i}{2}$

Example 11 Solve $\sqrt{5}x^2 + x + \sqrt{5} = 0$

Solution Here, the discriminant of the equation is

$$1^2 - 4 \times \sqrt{5} \times \sqrt{5} = 1 - 20 = -19$$

Therefore, the solutions are

$$\frac{-1 \pm \sqrt{-19}}{2\sqrt{5}} = \frac{-1 \pm \sqrt{19}i}{2\sqrt{5}}.$$

EXERCISE 5.3

Solve each of the following equations:

1. $x^2 + 3 = 0$ **2.** $2x^2 + x + 1 = 0$ **3.** $x^2 + 3x + 9 = 0$

4. $-x^2 + x - 2 = 0$ **5.** $x^2 + 3x + 5 = 0$ **6.** $x^2 - x + 2 = 0$

7. $\sqrt{2}x^2 + x + \sqrt{2} = 0$ **8.** $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

9. $x^2 + x + \dfrac{1}{\sqrt{2}} = 0$ **10.** $x^2 + \dfrac{x}{\sqrt{2}} + 1 = 0$

Miscellaneous Examples

Example 12 Find the conjugate of $\dfrac{(3-2i)(2+3i)}{(1+2i)(2-i)}$.

Solution We have , $\dfrac{(3-2i)(2+3i)}{(1+2i)(2-i)}$

$$= \frac{6+9i-4i+6}{2-i+4i+2} = \frac{12+5i}{4+3i} \times \frac{4-3i}{4-3i}$$

$$= \frac{48-36i+20i+15}{16+9} = \frac{63-16i}{25} = \frac{63}{25} - \frac{16}{25}i$$

Therefore, conjugate of $\dfrac{(3-2i)(2+3i)}{(1+2i)(2-i)}$ is $\dfrac{63}{25} + \dfrac{16}{25}i$.