

Sometimes it is convenient to construct a system of units so that all quantities can be expressed in terms of only one physical quantity. In one such system, dimensions of different quantities are given in terms of a quantity X as follows: [position] = [position] = $[X^\alpha]$; [speed] = $[X^\beta]$; [acceleration] = $[X^p]$; [linear momentum] = $[X^q]$; [force] = $[X^r]$. Then

This question has multiple correct options

A $\alpha + p = 2\beta$

B $p + q - r = \beta$

C $p - q + r = \alpha$

D $p + q + r = \beta$

(a, b) Given position, $L = [X^\alpha]$

Speed, $LT^{-1} = [X^\beta]$

Acceleration, $LT^{-2} = [X^p]$

Linear momentum, $MLT^{-1} = [X^q]$

Force, $MLT^{-2} = [X^r]$

$$\frac{\text{Position}}{\text{Speed}} = \text{time, } T = \frac{[X^\alpha]}{[X^\beta]} = X^{\alpha-\beta}$$

$$\text{Acceleration} = \frac{\text{Speed}}{\text{Time}} = \frac{X^\beta}{X^{\alpha-\beta}} = X^p$$

$$X^{\alpha-\beta+p} = X^\beta$$

$$\therefore \alpha + p = 2\beta$$

Hence option (a) is correct.

$$\text{Force} = \frac{\text{linear momentum}}{\text{time}}$$

$$[X^r] = \frac{[X^q]}{[X^{\alpha-\beta}]}$$

$$\Rightarrow r = q + \beta - \alpha \Rightarrow r = q + \beta - (2\beta - p)$$

$$\Rightarrow r = q - \beta + p \Rightarrow p + q - r = \beta$$

Hence option (b) is correct.