5. If α and β are the roots of the equation $x^2+px+2=0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2+2qx+1=0$, then $\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right)\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$ is equal to: [Main Sep. 03, 2020 (I)]

(a) $\frac{9}{4}(9+q^2)$ (b) $\frac{9}{4}(9-2^2)$

(c) $\frac{9}{4}(9+p^2)$ (d) $\frac{9}{4}(9-p^2)$

Solution: (d).

$\alpha.\beta=2$ and $\alpha+\beta=-p$ also $\frac{1}{\alpha}+\frac{1}{\beta}=-q \Rightarrow p=2q$

Now $\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right)\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$

$= \left[\alpha\beta+\frac{1}{\alpha\beta}-\frac{\alpha}{\beta}-\frac{\beta}{\alpha}\right]\left[\alpha\beta+\frac{1}{\alpha\beta}+2\right]$

$= \frac{9}{2}\left[\frac{5}{2}-\frac{\alpha^2+\beta^2}{2}\right] = \frac{9}{4}[5-(p^2-4)]$

$= \frac{9}{4}(9-p^2)$ $\left[\because \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta\right]$