2. If p and q are non-zero real numbers and $\alpha^3+\beta^3=-p$, $\alpha\beta=q$, then a quadratic equation whose roots are $\frac{\alpha^2}{\beta}, \frac{\beta^2}{\alpha}$ is:

(a) $px^2-qx+p^2=0$ (b) $qx^2+px+q^2=0$ [Main Online April 25, 2013]

(b) $px^2+qx+p^2=0$ (d) $qx^2-px+q^2=0$

Solution:

(b) · Given $\alpha^3+\beta^3=-p$ and $\alpha\beta=q$.

Let $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$ be the root of required quadratic equation.

So, $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}=\frac{-p}{q}$

and $\frac{\alpha^2}{\beta}\times\frac{\beta^2}{\alpha}=\alpha\beta=q$.

Hence, the required quadratic equation is:

$x^2-\left(\frac{-p}{q}\right)x+q=0$

$\Rightarrow x^2+\frac{p}{q}x+q=0 \Rightarrow qx^2+px+q^2=0$