A person measures the depth of a well by measuring the time interval between dropping a stone and receiving the sound of impact with the bottom of the well. The error in his measurement of time is $\delta T = 0.01$ seconds and he measures the depth of the well to be $L = 20$ meters. Take the acceleration due to gravity $g = 10\ ms^{-2}$ and the velocity of sound is $300\ ms^{-1}$. Then the fractional error in the measurement, $\delta L/L$, is closest to [Adv. 2017]

(a) 0.2% (b) 1% (c) 3% (d) 5%

(b) Depth of the well = $L = 20$ m

$\therefore$ Time taken by stone to reach the bottom of well,

$$t_1 = \sqrt{\frac{2L}{g}} \quad \left(\text{using, } L = \frac{1}{2}gt^2\right)$$

Time taken by impact sound to reach the person,

$$t_2 = \frac{L}{v}$$

Total time taken in the process is given by,

$$T = t_1 + t_2 = \sqrt{\frac{2L}{g}} + \frac{L}{v}$$

$$\therefore \quad \delta T = \sqrt{\frac{2}{g}}\frac{1}{2}\frac{\delta L}{\sqrt{L}} + \frac{\delta L}{v}$$

On substituting the given values we get,

$$\delta T = \frac{16}{300}\delta L$$

$\therefore$ Fractional error, $\frac{\delta L}{L} \times 100 = \frac{300}{16}\frac{\delta T}{L} \times 100$

$$= \frac{300}{16} \times \frac{0.01}{20} \times 100 \simeq 1\%$$