PROBLEM

. A function $g(x)$ is defined as $g(x) = \frac{1}{4} f(2x^2 - 1) + \frac{1}{2} f(1 - x^2)$ and $f'(x)$ is an increasing function. Then $g(x)$ is increasing in the interval

a. $(-1, 1)$

b. $\left(-\sqrt{\frac{2}{3}}, 0\right) \cup \left(\sqrt{\frac{2}{3}}, \infty\right)$

c. $\left(-\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}\right)$

d. none of these

SOLUTION

. **b.** $g'(x) = xf'(2x^2 - 1) - xf'(1 - x^2) = x(f'(2x^2 - 1) - f'(1 - x^2))$

$g'(x) > 0$

If $x > 0$, $2x^2 - 1 > 1 - x^2$ (as f' is an increasing function)

or $3x^2 > 2$ or $x \in \left(-\infty, -\sqrt{\frac{2}{3}}\right) \cup \left(\sqrt{\frac{2}{3}}, \infty\right)$

or $x \in \left(\sqrt{\frac{2}{3}}, \infty\right)$

If $x < 0$, $2x^2 - 1 < 1 - x^2$

or $3x^2 < 2$ or $x \in \left(-\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}\right)$ or $x \in \left(-\sqrt{\frac{2}{3}}, 0\right)$