PROBLEM

Let $g(x) = 2f\left(\frac{x}{2}\right) + f(2-x)$ and $f''(x) < 0 \ \forall \ x \in (0, 2)$.

Then $g(x)$ increases in

a. $(1/2, 2)$ **b.** $(4/3, 2)$

c. $(0, 2)$ **d.** $(0, 4/3)$

SOLUTION

d. We have $g'(x) = f'\left(\frac{x}{2}\right) - f'(2-x)$

Given $f''(x) < 0 \ \forall \ x \in (0, 2)$

So, $f'(x)$ is decreasing on $(0, 2)$.

Let $\frac{x}{2} > 2 - x$ or $f'\left(\frac{x}{2}\right) < f'(2-x)$.

Thus, $\forall \ x > \frac{4}{3}, g'(x) < 0$.

Therefore, $g(x)$ decreasing in $\left(\frac{4}{3}, 2\right)$ and increasing in $\left(0, \frac{4}{3}\right)$.