

JEE MAINS

PROBLEM

The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in [2007]

(a) $\left(0, \frac{\pi}{2}\right)$ (b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(c) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

SOLUTION

(d) Given $f(x) = \tan^{-1}(\sin x + \cos x)$

$$f'(x) = \frac{1}{1 + (\sin x + \cos x)^2} \cdot (\cos x - \sin x)$$

$$= \frac{\sqrt{2} \cdot \left(\frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x \right)}{1 + (\sin x + \cos x)^2}$$

$$= \frac{\left(\cos \frac{\pi}{4} \cdot \cos x - \sin \frac{\pi}{4} \cdot \sin x \right)}{1 + (\sin x + \cos x)^2}$$

$$\therefore f'(x) = \frac{\sqrt{2} \cos \left(x + \frac{\pi}{4} \right)}{1 + (\sin x + \cos x)^2}$$

if $f'(x) > 0$ then $f(x)$ is increasing function.

Hence $f(x)$ is increasing, if $-\frac{\pi}{2} < x + \frac{\pi}{4} < \frac{\pi}{2}$

$$\Rightarrow -\frac{3\pi}{4} < x < \frac{\pi}{4}$$

Hence, $f(x)$ is increasing when $n \in \left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$