Let $h(x) = f(x) - (f(x))^2 + (f(x))^3$ for every real number $x$. Then

*(1998 - 2 Marks)*

(a)  $h$ is increasing whenever $f$ is increasing
(b)  $h$ is increasing whenever $f$ is decreasing
(c)  $h$ is decreasing whenever $f$ is decreasing
(d)  nothing can be said in general.

SOLUTION

**(a, c)** We have

$$h'(x) = f'(x)[1 - 2f(x) + 3f(x))^2]$$

$$= 3f'(x)\left[(f(x))^2 - \frac{2}{3}f(x) + \frac{1}{3}\right]$$

$$= 3f'(x)[\{f(x) - 1/3\}^2 + 2/9]$$

Note that $h'(x) < 0$ whenever $f'(x) < 0$ and $h'(x) > 0$ whenever $f'(x) > 0$, thus, $h(x)$ increases (decreases) whenever $f(x)$ increases (decreases).