PROBLEM

If $f(x) = \begin{cases} 3x^2+12x-1, & -1\le x\le 2 \\ 37-x & 2<x\le 3 \end{cases}$ then: ***(1993 - 2 Marks)***

(a) $f(x)$ is increasing on $[-1, 2]$
(b) $f(x)$ is continues on $[-1, 3]$
(c) $f'(2)$ does not exist
(d) $f(x)$ has the maximum value at $x=2$

SOLUTION

(a, b, c) We are given that

$$f(x) = \begin{cases} 3x^2+2x-1, & -1\le x\le 2 \\ 37-x, & 2<x\le 3 \end{cases}$$

Then on $[-1, 2]$, $f'(x) = 6x+12$

For $-1\le x\le 2, -6\le 6x\le 12$

$\Rightarrow \quad 6\le 6x+12\le 24$

$\Rightarrow \quad f'(x)>0, \forall\, x \in [-1,2]$

$\therefore \quad f$ is increasing on $[-1, 2]$

Also $f(x)$ being polynomial for $x\in[-1,2)\cup(2,3]$

$f(x)$ is cont. on $[-1, 3]$ except possibly at

At $\quad x=2,$

$$\text{LHL} = \lim_{h\to 0} f(2-h) = \lim_{h\to 0} 3(2-h)^2 + 12(2-h) - 1$$
$$= 35$$

$$\text{RHL} = \lim_{h\to 0} f(2+h) = \lim_{h\to 0} 37 - (2+h) = 35$$

and $f(2) = 3.2^2 + 12.2 - 1 = 35$

LHL = RHL = $f(2)$

$\Rightarrow$ $f(x)$ is continuous at $x = 2$

Hence $f(x)$ is continuous on $[-1, 3]$

Again at $x = 2$

$$RD = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h\to 0} \frac{37 - (2+h) - 35}{h} = 1$$

$$LD = \lim_{h\to 0} \frac{f(2) - f(2-h)}{h}$$

$$= \lim_{h\to 0} \frac{35 - 3(2-h)^2 - 12(2-h) + 1}{h}$$

$$= \lim_{h\to 0} \frac{-3h^2 + 24h}{h} = 24$$

As $LD \neq RD$

$\therefore$ $f'(2)$ does not exist. Hence $f(x)$ can not have max. value at $x = 2$.