D **MCQs with One or More than One Correct**

PROBLEM

Let f and g be increasing and decreasing functions, respectively from $[0, \infty)$ to $[0, \infty)$. Let $h(x) = f(g(x))$. If $h(0) = 0$, then $h(x) - h(1)$ is *(1987 - 2 Marks)*

(a) always zero
(b) always negative
(c) always positive
(d) strictly increasing
(e) None of these.

SOLUTION

(a) Since g is decreasing in $[0, \infty)$

$\therefore$ For $x \geq y$, $g(x) \leq g(y)$ ....(1)

Also $g(x), g(y) \in [0, \infty)$ and f is increasing from $[0, \infty)$ to $[0, \infty)$.

$\therefore$ For $g(x), g(y) \in [0, \infty)$ such that $g(x) \leq g(y)$

$\Rightarrow$ $f(g(x)) \leq f(g(y))$, where $x \geq y \Rightarrow h(x) \leq h(y)$

$\Rightarrow$ h is decreasing function from $[0, \infty)$ to $[0, \infty)$

$\therefore$ $h(x) \leq h(0), \ \forall \ x \geq 0$

But $h(0) = 0$ (given)

$\therefore$ $h(x) \leq 0 \ \forall \ x \geq 0$ ...(2)

Also $h(x) \geq 0 \ \forall \ x \geq 0$ ...(3)

[as $h(x) \in [0, \infty)$]

From (2) and (3), we get $h(x) = 0, \ \forall \ x \geq 0$

Hence, $h(x) - h(1) = 0 - 0 = 0 \ \forall \ x \geq 0$