PROBLEM

For all $x \in (0,1)$ *(2000S)*

(a) $e^x < 1+x$

(b) $\log_e(1+x) < x$

(c) $\sin x > x$

(d) $\log_e x > x$

SOLUTION

(b) Let $f(x) = e^x - 1 - x$ then $f'(x) = e^x - 1 > 0$ for $x \in (0,1)$

$\therefore$ $f(x)$ is an increasing function.

$\therefore$ $f(x) > f(0), \forall\, x \in (0,1)$

$\Rightarrow$ $e^x - 1 - x > 0 \Rightarrow e^x > 1 + x$

$\therefore$ (a) does not hold.

(b) Let $g(x) = \log(1+x) - x$

then $g'(x) = \dfrac{1}{1+x} - 1 = -\dfrac{x}{1+x} < 0, \forall\, x \in (0,1)$

$\therefore$ $g(x)$ is decreasing on (0, 1) $\therefore x > 0$

$\Rightarrow$ $g(x) < g(0)$

$\Rightarrow$ $\log(1+x) - x < 0 \Rightarrow \log(1+x) < x$

$\therefore$ (b) holds. Similarly it can be shown that (c) and (d) do not hold.