

PROBLEM

The function  $f(x) = \sin^4 x + \cos^4 x$  increases if

*(1999 - 2 Marks)*

(a)  $0 < x < \pi/8$

(b)  $\pi/4 < x < 3\pi/8$

(c)  $3\pi/8 < x < 5\pi/8$

(d)  $5\pi/8 < x < 3\pi/4$

SOLUTION

(b) We are given  $f(x) = \sin^4 x + \cos^4 x$

$$\Rightarrow f'(x) = 4\sin^3 x \cos x - 4\cos^3 x \sin x$$

$$= -4 \sin x \cos x (\cos^2 x - \sin^2 x)$$

$$= -2 \sin 2x \cos 2x = -\sin 4x$$

Now for  $f(x)$  to be increasing function

$$f'(x) > 0 \Rightarrow -\sin 4x > 0 \Rightarrow \sin 4x < 0$$

$$\Rightarrow \pi < 4x < 2\pi \Rightarrow \frac{\pi}{4} < x < \frac{\pi}{2}$$

Since, If  $f(x)$  increasing on  $(\pi/4, \pi/2)$

$$\frac{\pi}{4} = \frac{4\pi}{8} > \frac{3\pi}{8}$$

It will be increasing on  $(\pi/4, 3\pi/8)$ .