<u>PROBLEM</u>

If $f(x) = \dfrac{x}{\sin x}$ and $g(x) = \dfrac{x}{\tan x}$, where $0 < x \leq 1$, then in this interval *(1997 - 2 Marks)*

(a) both $f(x)$ and $g(x)$ are increasing functions
(b) both $f(x)$ and $g(x)$ are decreasing functions
(c) $f(x)$ is an increasing function
(d) $g(x)$ is an increasing function.

<u>SOLUTION</u>

(c) We have $f(x) = \dfrac{x}{\sin x}, 0 < x \le 1$

$\Rightarrow \quad f'(x) = \dfrac{\sin x - x\cos x}{\sin^2 x}$

where $\sin^2 x$ is always +ve, when $0 < x \le 1$. But to check Nr., we again let

$h(x) = \sin x - x\cos x$

$\Rightarrow \quad h'(x) = x\sin x > 0$ for $0 < x \le 1 \Rightarrow h(x)$ is increasing

$\Rightarrow \quad h(0) < h(x)$, when $0 < x \le 1$

$\Rightarrow \quad 0 < \sin x - x\cos x$, when $0 < x \le 1$

$\Rightarrow \quad \sin x - x\cos x > 0$, when $0 < x \le 1$

$\Rightarrow \quad f'(x) > 0,\ x \in (0,1]$

$\Rightarrow \quad f(x)$ is increasing on $(0, 1]$

Again $g(x) = \dfrac{x}{\tan x}$

$\Rightarrow \quad g'(x) = \dfrac{\tan x - x\sec^2 x}{\tan^2 x}$, when $0 < x \le 1$

Here $\tan^2 x > 0$ But to check Nr. we consider

$p(x) = \tan x - x\sec^2 x$

$p'(x) = \sec^2 x - \sec^2 x - x.2\sec x.\sec x\tan x$

$\Rightarrow \quad p'(x) = -2x\sec^2 x\tan x < 0$ for $0 < x \le 1$

$\Rightarrow \quad p(x)$ is decreasing, when $0 < x \le 1$

$\Rightarrow \quad p(0) > p(x) \Rightarrow 0 > \tan x - x\sec^2 x$

$\therefore \quad g'(x) < 0$

Hence $g(x)$ is decreasing when $0 < x \le 1$.