PROBLEM

The function $f(x) = \dfrac{\ln\ (\pi + x)}{\ln\ (e + x)}$ is *(1995S)*

(a) increasing on $(0, \infty)$
(b) decreasing on $(0, \infty)$
(c) increasing on $(0, \pi/e)$, decreasing on $(\pi/e, \infty)$
(d) decreasing on $0, \pi/e)$, increasing on $(\pi/e, \infty)$

SOLUTION

(b) We have $f(x) = \dfrac{\ln(\pi + x)}{\ln(e + x)}$

$$\therefore \quad f'(x) = \frac{\left(\dfrac{1}{\pi + x}\right)\ln(e + x) - \dfrac{1}{(e + x)}\ln(\pi + x)}{[\ln(e + x)]^2}$$

$$= \frac{(e + x)\ln(e + x) - (\pi + x)\ln(\pi + x)}{(e + x)(\pi + x)(\ln(e + x))^2}$$

< 0 on $(0, \infty)$ since $1 < e < \pi$

$\therefore \quad f(x)$ decreases on $(0, \infty)$.