PROBLEM

Let the function $g:(-\infty, \infty) \rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ be given by $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$. Then, g is *(2008)*

(a) even and is strictly increasing in $(0, \infty)$

(b) odd and is strictly decreasing in $(-\infty, \infty)$

(c) odd and is strictly increasing in $(-\infty, \infty)$

(d) neither even nor odd, but is strictly increasing in $(-\infty, \infty)$

SOLUTION

(c) Given that $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$

$$\therefore g(-u) = 2\tan^{-1}\left(e^{-u}\right) - \frac{\pi}{2} = 2\tan^{-1}\left(\frac{1}{e^u}\right) - \frac{\pi}{2}$$

$$= 2\cot^{-1}\left(e^u\right) - \frac{\pi}{2} = 2\left[\frac{\pi}{2} - \tan^{-1}\left(e^u\right)\right] - \frac{\pi}{2}$$

$$= \pi - 2\tan^{-1}(e^u) - \frac{\pi}{2} = \frac{\pi}{2} - 2\tan^{-1}(e^u)$$

$= -g(u)$ $\therefore$ g is an odd function.

Also $g'(u) = \dfrac{2e^u}{1+e^{2u}} > 0$, $\forall u \in (-\infty, \infty)$

$\therefore$ g is strictly increasing on $(-\infty, \infty)$.