PROBLEM

If $f(x) = x^3 + bx^2 + cx + d$ and $0 < b^2 < c$, then in $(-\infty, \infty)$

(a) $f(x)$ is a strictly increasing function *(2004S)*

(b) $f(x)$ has a local maxima

(c) $f(x)$ is a strictly decreasing function

(d) $f(x)$ is bounded

SOLUTION

(a) $f(x) = x^3 + bx^2 + cx + d, 0 < b^2 < c$

$f'(x) = 3x^2 + 2bx + c$

Discriminant $= 4b^2 - 12c = 4(b^2 - 3c) < 0$

$\therefore$ $f'(x) > 0 \ \forall \ x \in R$

$\Rightarrow$ $f(x)$ is strictly increasing $\forall \ x \in R$