

49. The function  $f(x) = 4 \sin^3 x - 6 \sin^2 x + 12 \sin x + 100$  is strictly

(a) increasing in  $\left(\pi, \frac{3\pi}{2}\right)$                       (b) decreasing in  $\left(\frac{\pi}{2}, \pi\right)$

(c) decreasing in  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$                       (d) decreasing in  $\left(0, \frac{\pi}{2}\right)$

**Sol.** (b) We have,  $f(x) = 4 \sin^3 x - 6 \sin^2 x + 12 \sin x + 100$

$$\begin{aligned}\therefore f'(x) &= 12 \sin^2 x \cdot \cos x - 12 \sin x \cdot \cos x + 12 \cos x \\ &= 12 \cos x [\sin^2 x - \sin x + 1] \\ &= 12 \cos x [\sin^2 x + (1 - \sin x)]\end{aligned}$$

Now  $1 - \sin x \geq 0$  and  $\sin^2 x \geq 0$

$$\therefore \sin^2 x + 1 - \sin x > 0$$

Hence,  $f'(x) > 0$ , when  $\cos x > 0$  i.e.,  $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

So,  $f(x)$  is increasing when  $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

and  $f'(x) < 0$ , when  $\cos x < 0$  i.e.,  $x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

Hence,  $f(x)$  is decreasing when  $x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

Hence,  $f(x)$  is decreasing in  $\left(\frac{\pi}{2}, \pi\right)$