

22. Show that $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in $\left(0, \frac{\pi}{4}\right)$

Sol. We have, $f(x) = \tan^{-1}(\sin x + \cos x)$

$$\begin{aligned}\therefore f'(x) &= \frac{1}{1 + (\sin x + \cos x)^2}(\cos x - \sin x) \\ &= \frac{\cos x - \sin x}{1 + (\sin x + \cos x)^2}\end{aligned}$$

For $x \in \left(0, \frac{\pi}{4}\right)$, $\cos x > \sin x$

$$\therefore f'(x) > 0$$

$\therefore f(x)$ increases in $\left(0, \frac{\pi}{4}\right)$