

21. Show that for $a \geq 1$, $f(x) = \sqrt{3} \sin x - \cos x - 2ax + b$ is decreasing in R .

Sol. We have $f(x) = \sqrt{3} \sin x - \cos x - 2ax + b$

$$\begin{aligned}\therefore f'(x) &= \sqrt{3} \cos x + \sin x - 2a \\ &= 2 \left[\frac{\sqrt{3}}{2} \cdot \cos x + \frac{1}{2} \cdot \sin x \right] - 2a \\ &= 2 \left[\cos \frac{\pi}{6} \cdot \cos x + \sin \frac{\pi}{6} \cdot \sin x \right] - 2a = 2 \cos \left(\frac{\pi}{6} - x \right) - 2a\end{aligned}$$

If $f(x)$ is decreasing on R ,

$$\therefore f'(x) \leq 0 \text{ for all real } x$$

$$\Rightarrow 2 \cos \left(\frac{\pi}{6} - x \right) - 2a \leq 0 \text{ for all real } x$$

$$\Rightarrow a \geq \cos \left(\frac{\pi}{6} - x \right) \text{ for all real } x$$

$$\Rightarrow a \geq \max. \text{ value of } \cos \left(\frac{\pi}{6} - x \right)$$

$$\Rightarrow a \geq 1$$